You can replace some math skills with a calculator if you know how to operate the thing, but there’s no calculator for human interaction.

—Hal Sparks, actor

After twenty-five years as a mathematics educator, including many years of working directly with teachers, I decided to return to the classroom to teach one class of ninth-grade algebra for the year. I had been advocating for years what teachers should do in terms of raising expectations and shifting their teaching. I knew I was asking a lot of teachers, and I thought I needed to get back into the classroom for a reality check. That teaching experience not only humbled me but also taught me many lessons. One lesson in particular has stayed with me, and it continues to influence my thinking about what needs to take place in today’s mathematics classrooms. It involves a girl I call Crystal and her experience as a student of algebra and a student of arithmetic.

The Story of Crystal

My algebra class that year consisted of about twenty-five students. As I got to know my students, it became clear that they covered a wide spectrum of potential and challenges.

Crystal had been placed in this algebra class because her eighth-grade math teacher believed that she could handle algebra. But when I gave out a short survey at the beginning of the school year, asking students to tell me what they were good at in math and how they saw themselves as math students, Crystal’s response was, “Well, I’m okay at some things in math. But I can’t do fractions. I can’t add, subtract,
multiply, or divide them. So please don’t expect me to know fractions, because I don’t.” Crystal went on to say that she had noticed the “little blue calculators” at the back of the room (the TI Math Explorer calculators, which handled fractions). She said that if she could use one of those calculators, she thought she would be OK.

This was the first of what would be many tests for me regarding whether I could practice what I had been preaching. Would I allow a student to rely on a calculator, without too many restrictions, in order for her to study algebra? I decided to give it a try, and I checked out one of the calculators to Crystal for the school year.

After the first few months of school, we were well into the heart of algebra. Students were using algebra to represent situations, beginning to explore functions, solving equations, and so on. Crystal didn’t seem to be having any significant difficulties, and I honestly didn’t notice her achievement one way or the other. However, on one test in November, she did particularly well. As I returned the tests to the students, I said to her, “Crystal, you must have really studied hard for this test. You did a great job!”

Crystal responded, “Well, I really didn’t study all that hard. But, you know, I just kind of get it.”

As the school year progressed, it turned out that Crystal did, indeed, get it. I came to see that Crystal was one of the best algebraic thinkers I have ever known. She could represent essentially any word problem algebraically, from simple word problems to more complex problems calling for higher-level reasoning. And, provided she had her fraction calculator, once she had determined an equation, she could solve the equation, represent the solution graphically, and interpret the graph and the solution in terms of the original problem. In short, she could do algebra in ways teachers hope and dream for all of our students—even though she couldn’t do fractions. Wow.

The story could end right there and be a lesson to all of us: It is simply not the case that all students need to master all of the computation of arithmetic before they can tackle and succeed in algebra. Looking at the research (or lack of it) verifies my observation; there simply is no evidence that computational skill is the single path to success in algebra. But the story goes on. . . .

Time passed, and the school year began to wind down. I was starting to get nervous. My students had accomplished a lot during our year of studying algebra. But Crystal wasn’t the only one with lingering deficiencies in arithmetic. I knew that the following year my students would be moving on to Geometry and that other teachers would find out that some of my students couldn’t do some of their arithmetic. I worried about this a lot, and I tried to think of some way to address the problem. But I wasn’t willing to give up precious algebra class time.

Finally, I decided to tackle the arithmetic challenge head-on with student projects. My students had done a few outside projects during
the year, where they had used algebra on more in-depth problems than class time allowed. But instead of assigning another algebra project during our last grading period, I decided to take on computation. I gave my students a short test of computational skills. I asked them to identify in writing a computational skill that they wished they could improve—something that was getting in their way in algebra. I told them that they could work with their brothers, sisters, parents, friends, or me, or that they could use any of the different books in the classroom. For those few students who were confident (and proficient) in their computational skills, there were other options.

When I asked students to indicate what skill they were going to work on, Crystal wrote that she was going to learn how to add, subtract, multiply, and divide fractions. I thought her goal was ambitious, but I decided to see what would come of it. In deciding once and for all to conquer fractions, she made two wonderful and insightful comments. First, she wrote, \textit{The time has come!} Then she continued, \textit{I'm wasting too much time using my calculator.} She told me that she was tired of being a two-calculator girl, having to use her fraction calculator for computation and then her graphing calculator for graphing and other algebraic applications.

The way that students were to receive a grade for their computation project was that they had to sit with me one-on-one and convince me that they had learned what they said they were going to learn. When it was Crystal’s turn to meet with me, I said, “Now, before you show me what you have learned, I have to tell you something. You know I do a lot of talks for teachers, right?” She nodded. I continued, “I need to tell you that I’ve been talking about you. I call you Crystal, because I don’t use your real name, and I tell people that even though Crystal doesn’t do fractions, Crystal is one of the best algebraic thinkers I have ever known.” She grinned and got a little embarrassed, and then she proceeded to convince me, absolutely and positively, that she had learned how to add, subtract, multiply, and divide fractions. Wow.

\section*{There’s More to the Story}

This story has several epilogues worth sharing. The first occurred on the last day of school that year. I gave my students a year-end survey, and I asked them what they liked about the class, what they didn’t like, and what they thought I should tell teachers when I was giving presentations. As I read through the completed surveys, I smiled when I came across Crystal’s survey. In response to my last question, she had written, \textit{Tell them Crystal can do fractions.}

I saw Crystal from time to time after that year ended. I often wondered whether her fraction learning lasted. I knew that she had learned these skills before, and I knew that the procedures had not stuck with her. But every time I saw her, I was reluctant to ask her; after all,
it was a great story and I didn’t want to lose it. Finally, about two years later, I decided I had to know. So when I saw her the next time, I asked her if she still knew how to do fractions. She responded, “Yeah! And it’s coming in really handy in Algebra 2.” It turned out that Crystal had gone on to study Geometry and Algebra 2, and the next year, she was headed for Precalculus. She went on to a state university, where she finished her degree in nutrition in four years. She later continued her education through graduate school.

The final epilogue I’d like to share happened a few months after I finished that year of teaching. In August of that year, I was invited to be the keynote speaker to kick off the new school year for a small urban district. The superintendent had decided that this would be the year of mathematics, and that everyone in the district should come to the presentation by the “math lady.” Everyone meant that my audience included not only teachers and administrators but also bus drivers, cafeteria workers, custodians, and paraprofessionals, among others. I thought the best way to reach such a diverse audience was to tell some of my personal stories, including Crystal’s story. The presentation was well received, and when I finished, a young woman came up to me in tears. She said she was an aide in an elementary classroom. Then she looked at me and said, “I know I was Crystal. And nobody ever found me.” As she spoke, I couldn’t hold back my own tears. I think I apologized on behalf of the entire education system. On the long drive home after the presentation, I kept asking myself over and over how many Crystals I might have missed in my own teaching career.

What Can We Do?

Stories like Crystal’s can help us make better classroom decisions. As we learn lessons from our students, we can choose to act in ways in the future that support higher-level learning for all students. The lessons from Crystal’s story are numerous and perhaps obvious:

- Sometimes students can succeed at higher-level math, even without knowing everything we wish they knew about lower-level math. We owe it to all students to allow them to tackle the good stuff, and not reserve it just for some students.

- In the past, students without arithmetic proficiency would not have been able to deal with algebra. But in an age of calculators, we have a tool to allow all students to tackle algebra, regardless of computational gaps.

- Sometimes success at higher-level math—the good stuff—can motivate students to fill in their own deficiencies, to deal with the other stuff. Crystal finally learned fractions when she saw their usefulness in using algebra.

Technology is neither good nor bad in and of itself. It depends on how students use it and how teachers guide that use. Different students may use technology in different ways within the same classroom with positive results for their learning.

What we expect of students makes all the difference in the world, and we have a responsibility to help them achieve their fullest potential.

We never fully know the impact that we have on our students and on others.

Over the years, I have told Crystal’s story to hundreds of teachers and to diverse audiences. It always has a powerful effect. Some people even reconsider the limitations they place on students—limitations that seem to make sense at the time, but that may turn out to have unintended long-term negative consequences.

I ran into Crystal in a store not long ago. She was on her way to California for the next step in her life. I told her about the impact her story continues to have on so many others. She smiled at me and, with her usual sense of humor and acceptance of whatever life has to offer, said, “I do what I can.”

Reflection and Discussion

FOR TEACHERS

• What issues or challenges does this message raise for you? In what ways do you agree with or disagree with the main points of the message?

• What assumptions have you made about what students need to know before they can access higher-level mathematics? How critical do you think those assumptions are?

• What is your philosophy and practice regarding the role of calculators for your grade level? How open are you to modifying that philosophy based on what you learn?

• Do you have a Crystal story—something you’ve learned from one of your students that has reshaped your thinking?

• How can you recognize and support the Crystals in your classroom, who may not have mastered everything you wish they had mastered before they arrived at your class?

FOR FAMILIES

• What questions or issues does this message raise for you to discuss with your son or daughter, the teacher, or school leaders?
• How can you support your daughter’s or son’s progress into higher-level mathematics while helping with weaknesses in prior learning?
• How open are you to the possibility that there may be ways to use a calculator in support of students learning rigorous and challenging mathematics?

FOR LEADERS AND POLICY MAKERS

• How does this message reinforce or challenge policies and decisions you have made or are considering?
• How can you help teachers raise their expectations of all students, including those with gaps in their computational background?
• What policies, if any, does your school or district have in place regarding calculator use in mathematics classrooms? How well do these policies support student learning?
• How do you handle equity issues with respect to who can afford a calculator? How does the school or district ensure that every student has access to appropriate technology, including its availability at home?
• How can you help teachers learn appropriate ways of incorporating calculator use in pursuit of high-level mathematics?

RELATED MESSAGES

Faster Isn’t Smarter
• Message 31, “Do They Really Need It?,” looks at the critical role of teacher expectations on students’ mathematics learning.
• Message 20, “Putting Calculators in Their Place,” considers the role of calculators when developing computational skills within a balanced math program.
• Message 34, “Forgetting Isn’t Forever,” challenges the vision of mathematics as a linearly organized discipline, where each new topic depends on mastery of the previous topic.

Smarter Than We Think
• Message 3, “He Doesn’t Know His Facts,” recounts the story of a highly successful man in a mathematics-intensive field who succeeded in spite of not knowing his multiplication facts.

• Message 15, “You Can’t Do B If You Don’t Know A,” encourages us to question assumptions about what lower-level skills must be mastered before a student can be given the opportunity to explore higher-level skills.

• Message 1, “Smarter Than We Think,” illuminates the notion of intelligence and reinforces the importance of effort and perseverance.

• Message 17, “The Journey to Algebra,” looks at how we can help students learn to think algebraically at every grade level.

MORE TO CONSIDER

• A fourteen-minute video of Emma King presents a fascinating look at her life and work. She is noted for her work in mathematics and cosmology, after having overcome her dyslexia and difficulties dealing with numbers and computation. http://vega.org.uk/video/programme/301.

• *Fostering Algebraic Thinking: A Guide for Teachers, Grades 6–10* (Driscoll 1999) provides a comprehensive overview of how to help all students develop algebraic thinking and learn algebraic skills.

• *Algebra and Algebraic Thinking in School Mathematics: NCTM’s Seventieth Yearbook* (Greens and Rubenstein 2008) considers several issues related to the changing nature of algebra as an expectation for all students.

• *Mindset: The New Psychology of Success* (Dweck 2006) discusses the implications of a person’s mindset about intelligence on their school work and on their lives and offers insights for teachers about the value of providing challenging material to every student.

• “Ability and Mathematics: The Mindset Revolution That Is Reshaping Education” (Boaler 2013a) presents an overview of the impact of a growth mindset on mathematics teaching and helps us look for the talents of every student.

• The six principles underlying *Principles and Standards for School Mathematics* (NCTM 2000) present fundamental beliefs about equity, curriculum, teaching, learning, assessment, and technology. All six of these principles are relevant to Crystal’s story, but the discussions of equity, teaching, and technology are particularly pertinent and helpful.

• “Technology and Equity in Mathematics” (Seeley 1995) discusses issues related to appropriate technology use in support of equity for all students learning challenging mathematics. www.utdanacenter.org/mathtoolkit/support/best.php.

• *Algebra in a Technological World* (NCTM Addenda Series, Gr. 9–12; Heid et al. 1996) discusses the changing nature of algebra in today’s world, including ways to make algebra accessible to all students.