VIDEO CLIP

Introduction

Watch the video clip "Introduction." Talk with a friend or colleague; what do you see in the introduction that makes you most excited about *Beyond Invert & Multiply*?

Why This Resource?

Over the past several years fractions, as a topic in school mathematics, have received a great deal of attention. This is due, in large part, to growing research evidence that the majority of students enter middle school with limited understanding of fractions and, as a result, struggle with much of the mathematics content of middle school and beyond. With the Common Core State Standards for Mathematics, as well as other standards documents, the importance of understanding fractions as numbers has become a foundational aspect of fraction instruction in grades 3 and above. In addition, as students begin computing with fractions in subsequent grades, understanding fractions as numbers is central to helping students calculate with accuracy, efficiency, and understanding.

Why Is Fraction Computation So Hard?

Children have an intuitive sense about halving (as well as a very well-developed sense of fairness!) so their first encounter with fractions likely happens before they enter school. They may have shared a sandwich with a sibling or split a bag of candies with a friend. In these cases, children are concerned with making sure all parties get the same amount (or fair share), but once the partitioning is completed, it is unlikely that the term *half* is anything other than a label for the smaller portions, not a mathematical concept. The term *the larger half*, as in "She got the larger half!" is evidence of this phenomenon.

In the primary grades, students begin partitioning areas into fractional parts with consideration given to the parts being equal. Fraction names such as *halves*, *fourths*, and *eighths* are introduced, but are not yet labeled using numerical notation. Children as young as first grade can solve simple problems involving fraction computation when the problems are presented in a

meaningful context involving sharing (Carpenter 2014). They may draw the partitions to indicate each share, but typically do not know how to label the parts using fraction notation.

When students encounter formal fraction notation (usually in grade 3), they now must understand that fraction names such as *one-fourth* are not just a label for a part or portion, but describe a relationship between the part and the whole. In the fraction $\frac{1}{4}$, for example, students must understand that the digits 1 and 4 represent a relationship and must be considered differently than in the number 14. Depending on the context, $\frac{1}{4}$ may mean $\frac{1}{4}$ of one item or unit, $\frac{1}{4}$ of a group of items, or $\frac{1}{4}$ of a distance or interval (McNamara and Shaughnessy 2015). When students begin working on fraction computation, they must be able to abstract from the specifics of the context and operate on the fractions as numbers. For example, the computation involved in the following two problems is the same even though the answers represent very different lengths of time:

1. Shweta worked on her homework for $\frac{1}{4}$ of an hour before dinner and $\frac{1}{4}$ of an hour after dinner. What fraction of an hour did Shweta spend on her homework?
2. Lila spent 24 hours working on her science project. She spent $\frac{1}{4}$ of that time collecting data on Monday and $\frac{1}{4}$ of her time collecting data on Wednesday. She spent the rest of the time analyzing and writing up her results. What fraction of her time did Lila spend on data collection?

Both problems involve adding $\frac{1}{4}$ and $\frac{1}{4}$, however in Problem 1, the actual time Shweta spent on her homework is less than one hour and in Problem 2, the actual time Lila spent on her project is 12 hours. Being able to move flexibly between realistic contexts as shown here and decontextualized computation such as adding $\frac{1}{4} + \frac{1}{4}$ is an essential aspect of understanding fraction computation. Unless students are provided ample opportunity to develop an understanding of fractions as numbers, their success with problems such as these is unlikely.

Another challenge students have with fraction computation results from their previous experience with whole numbers. Students may be presented with three whole numbers such as 6, 4, and 24, as a *fact family*, and are often told that the largest number (in this case, 24) is the product in the multiplication problems $4 \times 6 = 24$ and $6 \times 4 = 24$ and the dividend in the division problems $24 \div 6 = 4$ and $24 \div 4 = 6$. It is not unusual for students to generalize this to mean that multiplication makes things bigger and division makes things smaller. This generalization (which makes good sense when working with whole numbers) can cause a lot of confusion when students encounter multiplication and division problems involving fractions. If students don't understand that $\frac{1}{2}$ is a number halfway between 0 and 1, they have little chance of understanding why multiplying 24 by $\frac{1}{2}$ results in a product less than 24 and dividing 24 by $\frac{1}{2}$ results in a quotient that is greater than 24.

In addition, it is not uncommon for students to learn rules and procedures for operating on fractions with little understanding, resulting in confusion over which procedure to apply when. For example, students may have been successful

multiplying across numerators and denominators and then (unsuccessfully) try the same approach with addition. Or students taught to "invert and multiply" or "keep, change, flip" to solve division problems with fractions may become confused about which part of the problem to invert, keep, change, or flip.

This resource and the accompanying video clips are designed to support you as you help your students develop a robust understanding of fractions and as they learn to calculate with accuracy, efficiency, and understanding. *Beyond Invert & Multiply* builds on the foundational understandings that are described in *Beyond Pizzas & Pies: 10 Essential Strategies for Developing Fraction Sense, Second Edition* (McNamara and Shaughnessy 2015) and applies them to situations involving computation. As in *Beyond Pizzas & Pies*, the approach is designed to build on students' fraction sense because, "Fraction sense is tied to common sense: Students with fraction sense can reason about fractions and don't apply rules and procedures blindly; nor do they give nonsensical answers to problems involving fractions" (McNamara and Shaughnessy 2015).

How This Resource Is Organized

Beyond Invert & Multiply is not necessarily intended to be read from cover to cover. Each section addresses different aspects of fraction computation. Different sections may have different levels of significance for you depending on your grade level and your students' particular needs. The order in which you read the sections doesn't matter; each section is written to stand alone.

This resource is organized into four parts:

- Part I addresses the meaning of fractions and understanding fractions as numbers. Chapter 1, Making Sense: Fractions as Numbers, contains the following:
    - *CCSS Connections:* The chapter begins with a connection to the relevant Common Core State Standards.
    - *Classroom Scenario:* The chapter addresses a common challenge that students encounter as they learn to operate with fractions. Some of these challenges may be tied to fraction notation, some may be connected to students' previous experience with whole numbers and fractions, and some may be the result of misapplying rules and procedures students learned when working with fractions. Each scenario is presented in the context of a fictional classroom episode in the third-grade classroom of Ms. Taylor, the fourth-grade classroom of Mrs. Ahmed, the fifth-grade classrooms of Ms. Chu and Mr. Gregory, and the sixth-grade classroom of Mr. Frank. While the exact episodes and students are fictional, the students' comments and struggles are taken from my work in classrooms as either teacher or researcher. The challenges are also not grade-level specific; you may find your sixth graders struggle in the same way as Ms. Taylor's third graders or your fourth graders may have some of the same difficulties as Ms. Chu's fifth graders.

- *What's the Math?* This section is intended to clarify the mathematics that is being addressed in the section.
- *What's the Research?* As teachers are asked to do more and more during the school day, it is imperative that we no longer continue with practices because "that's the way we've always done things." The research helps us to identify strategies, contexts, and representations that will ensure we get the most out of our instructional time, as well as those that may be problematic and/or limited.
- *Classroom Activities:* The chapter includes activities designed to help your students learn to calculate with accuracy, efficiency, and understanding. Materials lists, reproducibles, and examples of student projects are included. In addition, the accompanying video clips feature actual classroom footage of students and teachers engaged in several of the *Classroom Activities*. These activities are not meant to replace your current curriculum, but you may find that they will allow you to provide experiences for your students that help them further develop their ability to make sense of fraction computation.
- *Video Clips:* Chapters 1, 3, 6, 7, and 8 include video clips that were filmed in actual classrooms; see the tables on pages xxix–xxxii for a listing of clips by grade and chapter.
- *Wrapping It Up:* Each section ends with closing comments, study questions, and suggestions for additional resources when appropriate.

- Part II begins with Chapter 2, Developing Awareness: Addition and Subtraction Problem Types. This chapter is followed by two *Making Sense* chapters that address fraction addition as well as addressing fraction subtraction. The *Making Sense* chapters follow the same format as in Chapter 1.
- Part III begins with Chapter 5, Developing Awareness: Multiplication and Division Problem Types. The *Making Sense* Chapters 6 and 7 follow and address fraction multiplication as well as fraction division. These chapters follow the same format as in Chapter 1.
- Part IV begins with Chapter 8, Developing Awareness: Six Strategies for Fostering Student Talk About Fractions and addresses the challenging topic of mathematical discourse.

Get Started!

The importance of helping students make sense of fractions and fraction computation cannot be overstated. The scenarios and research findings presented in the following pages illustrate many of the challenges students without fraction sense face as they attempt to solve problems involving fraction computation. By providing opportunities for your students to investigate, discuss, revise, expand, and refine their understanding of fractions and fraction computation, you can prepare them for success with deeper application tasks they will encounter in middle school and beyond. In addition, this preparation will help them better understand, appreciate, and navigate the increasingly complex role that mathematics plays in the modern world.