Developing Algebraic Thinking Using Manipulatives
A Lesson Plan for Grades 3–5
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The lesson was adapted from the Math Solutions publications Lessons for Algebraic Thinking, Grades K–2 and Grades 3–5.

Algebra Content Standards
- Create numeric patterns that involve whole-number operations. (3-3.1)
- Apply procedures to find missing numbers in numeric patterns that involve whole-number operations. (3-3.2)
- Illustrate situations that show change over time as increasing. (3-3.4)

Lesson Process Standards
- Generate descriptions and mathematical statements about relationships. (3-1.4)
- Generalize connections between new mathematical ideas and related concepts that have been previously considered. (3-1.6)
- Use flexibility in mathematical representations. (3-1.7)

Lesson Objective (student version)
Today you will solve problems by building, extending, describing, and representing patterns that grow.

References
- Teaching Algebraic Thinking Grades K–2, by Marilyn Burns
  Chapter 7—Pattern Block Fish, pp. 79–88
  Chapter 12—Pattern Block Trees, pp. 157–170
- Teaching Algebraic Thinking Grades 3–5, by Marilyn Burns,
  Chapter 6—Iguanas: Building with Pattern Blocks, pp. 91–117

Materials
- 11 bags of pattern blocks: 9 squares, 9 trapezoids, 5 triangles, 5 hexagons
- 1 copy per student of Pattern Block Trees (recording sheet)
- 1 copy per student of Pattern Block Fish (recording sheet)
- Chart paper
- Markers
Materials (continued)

- Pictures of Elvis at 24 and at 80
  - www.8notes.com/elvis_presley.asp
  - www.strangepersons.com/content/item/11495.html
- Paper and pencil for each student
- Demonstration pattern blocks
- Demonstration charts of student recording sheets

Reviewed Vocabulary

hexagon, trapezoid, square, triangle, pattern

New Vocabulary

variable, constant, three-column chart, sketch, represent
Overview of Lesson

Students use pattern blocks as a tool to explore patterns that grow and solve problems. Using the contexts of trees and fish that grow in consistent and predictable ways, students build, extend, describe, and represent patterns. They record and analyze numerical data, link the data to the patterns generated with the pattern blocks, and make and verify predictions about how the trees and fish grow. Representations used in this lesson to show and analyze growth include pattern blocks, numerical data recorded in a three-column chart, and sketches.

Focus

1. In partners, students briefly explore the pattern blocks for these two purposes:
   - Review the names of the pattern blocks.
   - Practice several quick sketches of each of the blocks.

2. Introduce the task and model the idea of quick sketches. Allow five to seven minutes for students to name and sketch.

3. As a whole group, confirm the names of the shapes and record them on the board. Let students know they will be sketching the pattern blocks during the lesson, making recognizable but quick sketches to record their work.

4. As a whole class, briefly discuss patterns and a few examples students find in the room. Likely, these will be repeating patterns. Ask the following questions:
   - Q. What is a pattern?
   - Q. Where do you see patterns?
   - Q. How do you use patterns in math?

5. Share an example of how scientists and mathematicians use pattern to solve problems—face transformation. Include:
   - Background for Elvis;
   - Fans’ curiosity about what he would look like if he was still alive; and
   - Study of how the face changes and the use of patterns to predict what he would look like today.

   - Q. What is the same about Elvis’s face in both pictures?
   - Q. What is different or what has changed?

6. Transition from the real-life example to the lesson objective.
Engage

7. Introduce the task “Pattern Block Trees.” Model what the tree looks like on its 1st, 2nd, and 3rd birthday while students observe.

8. Ask students to build the tree on its 4th birthday and on its 5th birthday and check with their partners to see if they agree.

9. Volunteers build the tree at age 4 and at age 5 for the class to confirm how the tree grows.

10. Distribute recording sheet, “Pattern Block Trees.” Introduce the three-column chart to record how the tree is growing using numbers.

<table>
<thead>
<tr>
<th>Birthday</th>
<th>Which Blocks? How Many?</th>
<th>Total Number of Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>△ + △ + △</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>△ + △ + △</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>△ + △ + △</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>△ + △ + △</td>
<td>7</td>
</tr>
</tbody>
</table>

11. Model how to record the tree growth in the chart for birthdays 1, 2, and 3.
12. In partners, students fill in their own charts for birthdays 1, 2, and 3. In addition, they complete the chart for birthdays 4 and 5.

13. Ask students as a class to help complete the class chart as a check. In partners and as a whole class, discuss the following questions:
   Q. What patterns do you notice in the numbers?
   Q. What patterns do you notice when you look across each row?
   Q. What stays the same? What changes?

   **Teacher content note:**

   Students often see patterns vertically in the expanded chart. Listen to their observations and acknowledge the importance of those patterns. The push here is to help students also look across the rows to see how the numbers in the chart relate to birthday number/stage number.

   If students don’t “see” any relationships or patterns, the teacher’s role is to contribute an observation or two as examples for students. This usually prompts their observations. Some examples:
   • “I notice that regardless of the birthday number, there is always 1 triangle.”
   • “I notice that the number of trapezoids is the same as the birthday number.”
   • “The number of squares is the same as the birthday number.”
   • “The sum of the numbers in a row from the middle column equals the total column.”

   **Vocabulary note:**

   It is appropriate here to begin to use the term constant for the part of the pattern that stays the same and variable for the part of the pattern that changes.

14. Pose the following problem saying something like:
   “Let’s think about this tree when it’s 10 years old. You and your partner don’t have enough blocks partner to actually build the tree on its 10th birthday. But I do think you have enough information to figure out how many of each block you’ll need to build the tree on its 10th birthday.”

   Do two things with your partner:
   Decide how many of each block would be needed for the tree on its 10th birthday.
   Sketch the tree on your record sheet.

15. Have students confirm sketches and number of blocks with another partner pair.

16. Introduce the independent task.

**Explore**

17. Introduce “Pattern Block Fish.” Model the fish on its 1st, 2nd, and 3rd birthday. (See student record sheet for the task.)
18. Distribute the “Pattern Block Fish” recording sheet. Students work in partners to complete the task but each completes his or her own record sheet:
   • Build and sketch the 4th and 5th birthdays.
   • Complete the table for birthdays 1, 2, 3, 4, and 5.
   • Answer the two questions on the sheet.

19. Possible questions to focus and extend students’ learning during their independent work:
   Q. What stays the same? What is the constant?
   Q. What changes? What is the variable?
   Q. What patterns do you see?
   Q. How many blocks it would take to build the fish on its 8th birthday? 10th birthday?
   Q. What would it look like?
   Q. What if it took 14 blocks to build the fish? How old is the fish and what does it look like?

Assess

20. With student help, complete the class chart for pattern block fish from 1–10.
   Use questions from #19 to probe and focus student thinking.

21. Pose one or more of these questions for students to discuss with their partner and with the class:
   Q. What would we put in the chart for the 15th birthday? The 19th? The 100th?
   Depending on student responses, the following sentence frame may be posed for partner conversations:
   
   To build any pattern block fish, no matter how old, we would . . .

22. Review the lesson objective:

   Today you will solve problems by building, extending, describing, and representing patterns that grow.

   Talk with your partner about when in the lesson you:
   • Solved problems
   • Built and extended patterns that grow
   • Described and represented patterns that grow

   Ask students to complete this journal prompt:

   What did you learn today about solving problems using patterns that grow?

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