Praise for Beyond Pizzas & Pies . . .

The authors are right on—developing “fraction sense” is one of the most difficult tasks of new (or experienced) teachers. This book and the accompanying video clips provide a wonderful resource for helping teachers help their students become truly fluent with fractions.

—Barbara Reys, President
Curators’ Professor and Lois Knowles Faculty Fellow
University of Missouri, Columbia, Missouri

Beyond Pizzas & Pies is smart, accessible, and usable. McNamara and Shaughnessy assemble the key resources and insights teachers need to help students develop robust understanding of fractions. Clear explanations of the specific mathematical ideas and research on key challenges for learners are coordinated with focused instructional tasks and detailed guidance for their use in the classroom. The tasks develop complex mathematical proficiency by combining work on concepts with skill development and the ability to use representations and to reason about fractions.

—Deborah Loewenberg Ball, William H. Payne Collegiate Professor,
School of Education, University of Michigan

McNamara and Shaughnessy’s splendid work guides student understanding of fractions by engaging them in a relevant and thoughtful method. The authors bridge fundamental gaps between concept and application, using challenging and fun exercises that demonstrate these principals through visual and hands-on activities. Beyond Pizzas & Pies is an outstanding guide to a difficult unit on which so many mathematical concepts rely.

—Rebecca Gris, GATE teacher, grades 2–3
Boise Schools, Boise, Idaho

This book addresses not just what students should learn about fractions, but how children actually think, where that thinking comes from, and what teachers can do to make their instruction more effective. Each chapter is carefully structured to present the research base as well as classroom activities that can immediately be put to use. I can’t wait to use the book and video clips with my preservice teachers, since they summarize the most essential (and usable) information about the teaching and learning of fractions.

—Laurie Edwards, Professor of Education
Saint Mary’s College of California

Julie and Meghan have provided a multimedia resource rich in both mathematics content and pedagogical content. They clearly identify common student misconceptions about fractions and, even more importantly, offer practical ideas and activities to help deepen students’ essential understanding of fractions!

—Josh Rosen, Math Specialist
Dobbs Ferry School District, New York

This book and video clips lay out explicit strategies and activities for developing fraction understanding. The research, mathematics, and pedagogy are vividly brought to life through classroom scenarios and examples that will resonate with teachers.

—Carne Barnett-Clarke, Director
Math Pathways & Pitfalls, WestEd
Julie and Meghan’s multimedia resource forces educators to acknowledge that the traditional way of teaching fractions is limiting, lacking opportunity for higher-level application. They offer concrete solutions that can be easily integrated into any classroom curriculum.

—Lindsay Phillips, eighth-grade teacher
Portsmouth Middle School, Portsmouth, Rhode Island

_Beyond Pizzas & Pies_ expertly presents the essential concepts that third-, fourth-, and fifth-grade teachers must fully understand so they can teach students to develop a deep conceptual understanding of fractions. By presenting ten critical strategies for developing fraction sense, McNamara and Shaughnessy provide a tightly scaffolded plan for both teachers and students that is easy to follow and goes well beyond the usual teacher manuals and math texts commonly used in upper elementary school classrooms.

—Maggie Riddle, Director of Schools,
Berkeley Unified School District, California

McNamara and Shaughnessy understand the difficulty of fractions in mathematics education and masterfully weave together classroom vignettes, the latest in education research, mathematics, and classroom lessons in easy-to-follow chapters that will challenge your assumptions about how to teach this critically important topic. This multimedia resource is a goldmine of information delivered in simple, straightforward language.

—Jennifer M. Langer-Osuna, PhD, Assistant Professor
Stanford University, Stanford, California

Using these activities has increased my students' deep understanding of fractions and their math confidence, while decreasing their confusion when we discuss decimals. I needed this resource when I started teaching thirty years ago!

—Morri Spang, fifth-grade teacher,
San Pedro St. Elementary School, Los Angeles, California

McNamara and Shaughnessy have created a gem of a resource for teaching the very difficult content area of fractions. This book and the accompanying video clips are guaranteed to support teachers in developing deeper understandings of the complexities involved in reasoning about and computing with fractions, so that they in turn can support the developing understandings of their students.

—Mary Q. Foote, Assistant Professor of Mathematics Education
Queens College, CUNY, New York

Bravo to the clever authors of _Beyond Pizzas & Pies_ for serving both teachers and students a practical menu with fresh recipes of applicable strategies that will help us think outside the pie. The explanations backed by research stimulated and altered my own math sense, empowering my instruction with more purpose and comprehension. My fifth graders will be challenged and charmed as we all deepen our fraction knowledge.

—Karen L. Thompson, fifth-grade teacher
Farrand Elementary, Plymouth, Michigan
Beyond Pizzas & Pies
SECOND EDITION

10 Essential Strategies for Supporting Fraction Sense

Julie McNamara
Meghan M. Shaughnessy

Foreword by Francis (Skip) Fennell

Math Solutions
Sausalito, California, USA
A Message from Math Solutions

We at Math Solutions believe that teaching math well calls for increasing our understanding of the math we teach, seeking deeper insights into how students learn mathematics, and refining our lessons to best promote students’ learning.

Math Solutions shares classroom-tested lessons and teaching expertise from our faculty of professional development consultants as well as from other respected math educators. Our publications are part of the nationwide effort we’ve made since 1984 that now includes

- more than five hundred face-to-face professional development programs each year for teachers and administrators in districts across the country;
- professional development books that span all math topics taught in kindergarten through high school;
- videos for teachers and for parents that show math lessons taught in actual classrooms;
- on-site visits to schools to help refine teaching strategies and assess student learning; and
- free online support, including grade-level lessons, book reviews, inservice information, and district feedback, all in our Math Solutions Online Newsletter.

For information about all of the products and services we have available, please visit our website at www.mathsolutions.com. You can also contact us to discuss math professional development needs by calling (800) 868-9092 or by sending an email to info@mathsolutions.com.

We’re always eager for your feedback and interested in learning about your particular needs. We look forward to hearing from you.

Math Solutions
FOUNDED BY MARILYN BURNS

SCHOLASTIC
For Haley, Kelsey, and Rick.
You amaze me daily.
—JM

For my parents, Mike and Barbara, and for the elementary school teachers and students who inspired the writing of this book.
—MMS
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If there is a foundational building block for understanding fractions, considering fractions as numbers gets my vote. The Common Core State Standards for Mathematics’ focus on fractions as number within the Number and Operations—Fractions content domain underscores this importance. And, yes as learners experience, understand, and develop proficiency with fractions a sense of number emerges: Fraction Sense, the desired prerequisite for developing understanding and proficiency, even fluency, with operations involving fractions and decimals, and the readiness for middle grade work with ratio and proportion.

The second edition of *Beyond Pizzas & Pies*, like the first edition, addresses a major challenge for every third- through fifth-grade teacher I have ever met. Fractions—defined as \( \frac{a}{b} \) fractions, decimals, and related percents—are important building blocks for higher-level mathematics. This edition also includes video clips of students filmed in actual classrooms who are engaged in a sampling of the book’s activities, thus providing more professional development opportunities for this amazing resource. I could go on and on here, but allow me to provide just a few indicators as to the importance of fractions as numbers—considering partitioning/sharing, equivalence, use of varied representations, and comparing and ordering fractions as elements of these prerequisites of fraction sense and necessary prior to extensive work with operations involving “these” numbers (\( \frac{a}{b} \) fractions and decimals).

- In 2008, more than 740 teachers of algebra responded to a National Mathematics Advisory Panel survey about the preparation of their students for algebra. Teachers identified rational numbers as a major area of concern, and the Panel listed proficiency with fractions as a major goal for preK–8 mathematics education.
- Only 50 percent of eighth-grade students successfully arranged \( \frac{3}{7}, \frac{1}{12}, \) and \( \frac{5}{8} \) from least to greatest (NAEP 2004).
- Only 29 percent of seventeen-year-old students translated 0.029 as \( \frac{29}{1000} \) (NAEP 2004).
- Finally, while we all have our favorite fraction stories, consider this one. I once asked a fifth grader where I might place \( \frac{9}{5} \) on a number line, the student insisted that this could not be done, “because nine-fifths is more than one.” By implication, number lines end at 1!

As we consider the “curricular fit” of \( \frac{a}{b} \) fractions, decimals, percent, and their middle grade extensions involving ratio, rate, and proportion, we must attend to the need for teachers to address foundational fraction concepts. Check that off—this book does continues to do this—and the access to classroom videos makes it even better! Take a look at each chapter. First, note how fractions are approached with early opportunities involving partitioning and sharing (which, by the way, is far too frequently disregarded in typical curricular expectations). Next, see the progression toward the importance of equivalence, comparing, and ordering, and making sense of fractions and their more frequently used equivalent: decimals. In addition, this amazing resource examines contexts where,
yep, sometimes you’d rather have \( \frac{1}{3} \) than \( \frac{2}{3} \). And it also considers the appropriateness of particular representations, a particularly important topic for those implementing the Common Core State Standards for Mathematics (CCSSM).

Importantly, Julie McNamara and Meghan Shaughnessy have found a way not only to insert important research into this work but also to blend it with the issues addressed. And this is done on a consistent basis. I am a firm believer that we must all attend to research, and this book exemplifies the phrase linking research to practice. Even the title grabs me: you’d better believe that foundational work with fractions is beyond the overused references of pizzas and varied and sundry other pies!

At this writing, schools are in the implementation stage of the Common Core State Standards for Mathematics. However, whether you are following the Common Core, your own state’s standards or your district’s adopted textbook, this resource has got you covered. All fractions are important, and *Beyond Pizzas & Pies* examines the foundations necessary for all students.

I am going to continue to use this book—a lot (I keep one copy at home and one at the office for easy access)! It will be at my side as I think about fractions as critically important mathematical knowledge for teaching. *Beyond Pizzas & Pies* and the accompanying DVD will also help me think of ideas for professional development, for teaching, and for continuing the quest we all have for our students: developing number sense and ensuring that number sense extends to fractions—Fraction Sense. Thanks, Julie and Meghan.

—Francis (Skip) Fennell, L. Stanley Bowlsbey Professor of Education & Graduate and Professional Studies, McDaniel College, Westminster, Maryland, and Past President, National Council of Teachers of Mathematics Project Director: Elementary Mathematics Specialists and Teachers Leaders Project, MathSpecialists.org
Why This Resource?

If you ask an upper elementary teacher to name the most challenging mathematics topic for students, chances are you’ll hear, “Fractions.” Many middle school teachers find they have to review fraction concepts and operations before their students can successfully tackle algebra. We’ve spoken to countless adults who will unabashedly claim they hate fractions and always have. Sadly, students without a strong understanding of fractions find it difficult to progress very far in mathematics. In fact, the California State University/University of California Mathematics Diagnostic Testing Project Workgroup has found that there is a strong positive correlation between students’ understanding of fractions and their overall success in mathematics (Gomez 2009). According to the Final Report of the National Mathematics Advisory Panel (2008), understanding fractions is a “foundation skill essential to success with algebra” (3). The report also states that completion of Algebra II correlates highly with future academic success as well as earning potential.

This book and the accompanying video clips are intended to support you as you help your students develop their fraction sense. Fraction sense implies a deep and flexible understanding of fractions that is not dependent on any one context or type of problem. Fraction sense is tied to common sense: Students with fraction sense can reason about fractions and don’t apply rules and procedures blindly; nor do they give nonsensical answers to problems involving fractions. Because we believe that fraction sense is an essential component for students’ success with fraction operations, this book and the accompanying video clips focus primarily on helping students build foundational fraction concepts.
Why Is Understanding Fractions So Hard?

In order to support your goal to help students develop what we call “Fraction Sense,” we first need to identify some of the reasons that so many children and adults have such difficulty with fractions.

Helping students develop a deep and flexible understanding of fractions is a complex endeavor. In order for students to understand fractions and fraction notation, they must be able to think about numbers in a different way than when they are working with whole numbers. For example, instead of viewing a number such as 34 as representing a specific quantity, when the same digits (3 and 4) are used in the number \( \frac{3}{4} \), the digits 3 and 4 represent a relationship. In addition, students need to consider the context in which the number \( \frac{3}{4} \) occurs. While the relationship between the numerator 3 and the denominator 4 doesn’t change across contexts, the way the fraction is represented does.

When considering \( \frac{3}{4} \) as a number, the 3 represents three one-fourths, and the whole, or unit, is one.

Research identifies several factors that likely contribute to students’ difficulties with fractions, including but not limited to:

- The way that fractions are written (Ball 1993; Davydov and Tsvetkovich 1991; Lamon 2007; National Research Council 2001; Smith 2002);
- Classroom practices designed to help students make sense of fraction values and notation that inadvertently mask the meaning of fractions (Armstrong and Larson 1995; Davydov and Tsvetkovich 1991; Lamon 2007; Mack 1990, 1995);
- Students’ overreliance on whole number knowledge (Mack 1990; Saxe et al. 2007); and
- The many meanings of fractions, such as measure and ratio (Lamon 2001; National Research Council 2001).
When considering $\frac{3}{4}$ as part of an area, the 3 represents three replications of the area that is one-fourth of the whole.

![Diagram of a rectangle divided into four equal parts, with three parts shaded to represent $\frac{3}{4}$]

When $\frac{3}{4}$ is considered as a measure, the 3 is three iterations of the distance that is one-fourth of the whole.

![Diagram of a line divided into quarters with $\frac{3}{4}$ shown as three segments]

When $\frac{3}{4}$ is considered as part of a set, the 3 could mean three items, six items, twenty-four items, and so on, depending on the size of the entire set.

![Diagram of sets of objects with $\frac{3}{4}$ represented]

**How This Resource Is Organized**

This book is not necessarily intended to be read from cover to cover. Each chapter discusses one common dilemma that students have with fractions and includes classroom strategies and activities for preventing and addressing these dilemmas. Different chapters may have different levels of significance for you, depending on your grade level and your students’ particular needs. The order in which you read the chapters doesn’t matter; each chapter is written to stand alone.

Each of the eight chapters follows the same format:

- **CCSS Connections:** Chapters open with connections to the relevant Common Core State Standards.
- **Classroom Scenario:** Each chapter identifies a dilemma that students encounter as they begin their formal study of fractions. Some of these
dilemmas may be tied to fraction notation, some may be connected to
students’ previous experiences with fractions, and some may be the
result of misapplying rules and procedures students learned when work-
ing with whole numbers. Each dilemma is presented in the context of a
fictional classroom episode in the third-grade classroom of Mr. Burnett,
the fourth-grade classroom of Ms. Alvarez, or the fifth-grade classroom
of Ms. Chu. While the exact episodes and students are fictional, the stu-
dents’ comments and struggles are taken from our work in classrooms
as either teacher or researcher. The dilemmas are also not grade-level
specific; you may find that your fifth graders struggle in the same way
as Mr. Burnett’s third graders or your third graders may have the same
difficulties as Ms. Alvarez’s fourth graders.

- **What’s the Math?** This section is intended to clarify the mathematics that
is being addressed in each chapter.

- **What’s the Research?** As teachers are asked to do more and more during
the school day, it’s imperative that we no longer continue with practices
because “that’s the way we’ve always done things.” The research helps
us to identify those strategies, contexts, and representations that may
be problematic and/or limited, as well as those that will ensure we get
the most out of our instructional time.

- **Classroom Activities:** Each chapter includes activities designed to help
your students develop their fraction sense. Materials lists, reproducibles,
and examples of student projects are included. In addition, the accom-
panying video clips feature actual classroom footage of students and
teachers engaged in several of the Classroom Activities. These activities
are not meant to replace your current curriculum, but you may find
that they will allow you to provide experiences for your students that
help them further develop their fraction sense.

- **Wrapping It Up:** Each chapter ends with closing comments, study
questions, and suggestions for additional resources when appropriate.

- **Video Clips:** Chapters 2, 3, 5, and 8 include video clips that were filmed
in actual classrooms; see the tables on pages xxxi–xxxiv for a listing of
clips by grade and chapter.

### What’s New to the Second Edition?

The second edition of *Beyond Pizzas & Pies* is updated to reflect the authors’ most
current thinking and research. In addition, the second edition now includes 30
video clips filmed in actual classrooms. Clips range from 1 minute to 6 minutes
in length with a total viewing time of approximately 70 minutes.
The 10 Essential Strategies

Following is an overview of the eight main ideas discussed in each of the chapters plus two additional strategies that permeate throughout the chapters. These two final strategies are not specific to fraction instruction. Instead, they are essential instructional techniques regardless of the content you are teaching. We feel, however, that they are particularly important to consider when helping students develop fraction sense because of the complexity of the topic and the many challenges students and teachers often face when it comes to the learning and teaching of fractions. We hope that you will find these to be valuable additions to your repertoire of teaching strategies.

Strategy #1: Provide opportunities for students to work with irregularly partitioned, and unpartitioned, areas, lengths, and number lines.

By providing opportunities for students to create partitions and reason about unequally partitioned shapes, you can help them develop a deep and flexible understanding of part–whole relations. As we discuss in Chapter 1, these kinds of experiences can help students move beyond a shallow understanding of part–whole relations that is based on merely counting parts to one that is based on truly understanding the relationship between the shaded part and

Ten Essential Strategies for Supporting Fraction Sense

1. Provide opportunities for students to work with irregularly partitioned, and unpartitioned, areas, lengths, and number lines. (Chapter 1)

2. Provide opportunities for students to investigate, assess, and refine mathematical “rules” and generalizations. (Chapter 2)

3. Provide opportunities for students to recognize equivalent fractions as different ways to name the same quantity. (Chapter 3)

4. Provide opportunities for students to work with changing units. (Chapter 4)

5. Provide opportunities for students to develop their understanding of the importance of context in fraction comparison tasks. (Chapter 5)

6. Provide meaningful opportunities for students to translate between fraction and decimal notation. (Chapter 6)

7. Provide opportunities for students to translate between different fraction representations. (Chapter 7)

8. Provide students with multiple strategies for comparing and reasoning about fractions. (Chapter 8)

9. Provide opportunities for students to engage in mathematical discourse and share and discuss their mathematical ideas, even those that may not be fully formed or completely accurate.

10. Provide opportunities for students to build on their reasoning and sense-making skills about fractions by working with a variety of manipulatives and tools, such as Cuisenaire rods, Pattern Blocks, Fraction Kits, and ordinary items from their lives.
the whole, the distance from 0 in relation to the unit distance on a number line, or the relationship between rods of different lengths.

**Strategy #2: Provide opportunities for students to investigate, assess, and refine mathematical “rules” and generalizations.**

It is not uncommon for children to misapply generalizations as they attempt to make sense of new and complex material. Helping them question and refine generalizations and strategies is extremely important in supporting students’ development as mathematical sense-makers. In Chapter 2, we describe some activities you can use in your classroom.

**Strategy #3: Provide opportunities for students to recognize equivalent fractions as different ways to name the same quantity.**

Students should understand that equivalent fractions—such as \( \frac{8}{12} \), \( \frac{4}{6} \), and \( \frac{2}{3} \)—represent precisely the same point on the number line, and the differences in notation (how the fractions are written) are merely a matter of the value of the denominator. As we describe in Chapter 3, students often do not understand that equivalent fractions are multiple ways to name one quantity.

**Strategy #4: Provide opportunities for students to work with changing units.**

We need to help students understand that an object such as a triangle pattern block can be one-third of one thing (a trapezoid) and one-sixth of another (a hexagon). Activities like the ones we describe Chapter 4, which use materials such as Cuisenaire rods and Pattern Blocks and designate different items as the whole (or unit) can support students’ understanding that a fraction is not a name for a given block but a relationship between the block and the whole.

**Strategy #5: Provide opportunities for students to develop their understanding of the importance of context in fraction comparison tasks.**

Fractions are representations of quantities, and these quantities are measured in relation to a unit (or a whole). The meaning of \( \frac{2}{3} \) is determined in part by the size of the unit. As we describe in Chapter 5, students need opportunities to think about the importance of context in fraction comparison problems.

**Strategy #6: Provide meaningful opportunities for students to translate between fraction and decimal notation.**

Fraction and decimal notation are two different notational systems for rational numbers. Frequently, students do not see that a fraction and a decimal are merely two different ways to name a quantity. As we describe in Chapter 6, providing students with opportunities to work simultaneously with fraction and decimal notations supports their development of fraction sense.
Strategy #7: Provide opportunities for students to translate between different fraction representations.

In the elementary grades, students are introduced to multiple representations for fractions, including shaded parts of areas, parts of sets, and points on the number line. In Chapter 7 we discuss the importance of asking students to translate between different fraction representations. When students translate, they are forced to consider the features of the representation that are representation-specific, as well as those that have mathematical meaning beyond the representation.

Strategy #8: Provide students with multiple strategies for comparing and reasoning about fractions.

Students often rely on a common denominator strategy for comparing fractions, even when other strategies are more convenient or efficient. As we describe in Chapter 8, you can support your students in deepening their fraction sense by providing opportunities for them to use benchmarks to reason about fraction value, and by focusing their attention on the relationships between numerators and denominators (not just numerators or denominators).

Strategy #9: Provide opportunities for students to engage in mathematical discourse and share and discuss their mathematical ideas, even those that may not be fully formed or completely accurate.

In addition to providing opportunities for students to learn from their classmates, to refine their thinking by explaining to another person, and to use mathematical language in a meaningful way, classroom discussions can provide you with invaluable insights into what students do and do not understand about a given topic.

Strategy #10: Provide opportunities for students to build on their reasoning and sense-making skills about fractions by working with a variety of manipulatives and tools, such as Cuisenaire rods, Pattern Blocks, Fraction Kits, and ordinary items from their lives.

The more opportunities that students have to apply their fraction knowledge to solve problems involving different materials, contexts, settings, and relationships, the deeper and more flexible their understanding of fraction concepts will become.
Get Started!

The importance of helping students develop a deep and flexible understanding of foundational fraction concepts, such as those discussed in this book, cannot be overstated. The scenarios and research findings presented in the followings chapters illustrate many of the challenges students without fraction sense face as they attempt to solve fraction problems. By providing opportunities for your students to investigate, discuss, revise, expand, and refine their understanding of fractions, you can prepare them for success with fraction comparison and computation. This preparation will not only help them with fraction tasks they encounter in school, but it will also help them better appreciate and understand the important role fractions play in their world.

See Also

Beyond Invert & Multiply: Making Sense of Fraction Computation, Grades 3–6
A Multimedia Professional Learning Resource

This resource builds on the foundational understandings described in Beyond Pizzas & Pies and applies them to situations involving fraction computation. Throughout the resource connections are made to the “Ten Essential Strategies for Supporting Fraction Sense” previously listed.
A Letter from Math Coach Lori MacDonald

I cannot recall a time in my professional life when I’ve been more grateful for a resource. Teachers in Berkeley have been working hard learning the Common Core standards, paying special attention to the new emphasis on fractions. The standards seem logical, but we have many questions and some concerns as we enter our second year teaching them. Julie McNamara and Meghan Shaughnessy, in an immeasurably helpful way, have anticipated our situation and designed *Beyond Pizzas & Pies, Second Edition: 10 Essential Strategies for Supporting Fraction Sense* to lead us, step-by-important-step, into greater understanding of the standards and ways we might teach them effectively.

Many teachers are wondering, even after studying the standards carefully, “How am I supposed to teach something I don’t yet fully understand?” My answer is, read *Beyond Pizzas & Pies*. Here’s a quick look at how each section provides invaluable support.

**Classroom Scenario**

In the “Classroom Scenario” sections of *Beyond Pizzas & Pies*, the authors get right inside our heads. Through their deft analysis of teachers' thinking process, Julie and Meghan assure the reader that they understand our need to be led through a guided process to gain greater understanding. If we could have mastered the standards in one reading, we would have. Julie and Meghan begin each chapter with us in the classroom as we struggle with knowing how best to teach these unfamiliar concepts.

**What’s the Math?**

The authors then begin a short but effective section of direction instruction, “What’s the Math?,” and we are listening, because we know Julie and Meghan get it. Having then acquired a better understanding of the concept at hand, we are pushed to consider what the research says. Where are the typical misunderstandings among students? Among teachers? What has been shown to be most effective in terms of framing the concept?

**Classroom Activities**

Then, once our trust has been earned and our understanding increased, right about the time we’re thinking, “This is great, but I don’t have time to translate this into lessons,” *Beyond Pizzas & Pies* gives us “Classroom Activities,” replete with reproducibles and video clips demonstrating how to teach the concept.

**Wrapping It Up**

In “Wrapping It Up,” we are asked questions that show us how far we’ve come and challenge us to set concrete goals for implementing what we’ve learned.
Throughout the book, the reader feels as if she has a personal math guide. This is not an academic book, simply offering some good ideas on how you might approach teaching fractions. It is a systematic walk through of the Common Core State Standards for Mathematics. In the middle of explaining a new approach taken in the CCSSM, the authors remind us of how previous standards addressed similar concepts. By reminding us of how we previously understood (and probably taught) various concepts, our learning of the new standards is greatly facilitated. No ideas in this book are offered in a vacuum; Julie and Meghan create a context for all they discuss.

What a gift *Beyond Pizzas & Pies* is to teachers. Personal. Practical. Professional. I want every teacher and coach in Berkeley Unified to have a copy of this resource and to wear it out: mark it, bend its spine, and mine it for all its riches and classroom videos.

—Lori MacDonald, Coach, K–5 mathematics Berkeley Unified School District, California
Summary of Connections with the Common Core State Standards

The following connections are also featured at the beginning of each of the corresponding chapters; they are included here as well for quick reference.

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<td>1</td>
<td><strong>Grade 3</strong>&lt;br&gt;Number &amp; Operations—Fractions&lt;br&gt;<em>Develop understanding of fractions as numbers.</em>&lt;br&gt;3NF.A.1: Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.&lt;br&gt;3NF.A.2: Understand a fraction as a number on the number line; represent fractions on a number line diagram.&lt;br&gt;Geometry&lt;br&gt;<em>Reason with shapes and their attributes.</em>&lt;br&gt;3.G.A.2: Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.</td>
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<td>2</td>
<td><strong>Grade 4</strong>&lt;br&gt;Number &amp; Operations—Fractions&lt;br&gt;<em>Extend understanding of fraction equivalence and ordering.</em>&lt;br&gt;4.NF.A.1: Explain why a fraction a/b is equivalent to a fraction (n x a)/(n x b) by using visual models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.&lt;br&gt;4.NF.A.2: Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols &lt;, =, or &lt;, and justify the conclusions, e.g., by using a visual fraction model.</td>
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<td><strong>Grade 4</strong>&lt;br&gt;Number and Operations—Fractions&lt;br&gt;<em>Extend understanding of fraction equivalence and ordering.</em>&lt;br&gt;4.NF.A.1: Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.&lt;br&gt;4.NF.A.2: Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $&gt;$, $=$, or $&lt;$, and justify the conclusions, e.g., by using a visual fraction model.</td>
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<td><strong>Grade 5</strong>&lt;br&gt;Number and Operations—Fractions&lt;br&gt;<em>Use equivalent fractions as a strategy to add and subtract fractions.</em>&lt;br&gt;5.NF.A.1: Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.&lt;br&gt;5.NF.A.2: Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fractions models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.</td>
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<p>| Grade 3 | Number and Operations—Fractions&lt;br&gt;<em>Develop understanding of fractions as numbers.</em>&lt;br&gt;3.NF.A.1: Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into $b$ equal parts; understand a fraction $a/b$ as the quantity formed by a parts of size $1/b$. |
|         | <strong>Geometry</strong>&lt;br&gt;<em>Reason with shapes and their attributes.</em>&lt;br&gt;3.GA.2: Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. |
|         | <strong>Grade 4</strong>&lt;br&gt;Number and Operations—Fractions&lt;br&gt;<em>Extend understanding of fraction equivalence and ordering.</em>&lt;br&gt;4.NF.1: Explain why a fraction $a/b$ is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. |</p>
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<td><strong>Grade 5</strong>&lt;br&gt;Number and Operations—Fractions&lt;br&gt;Apply and extend previous understandings of multiplication and division.&lt;br&gt;&lt;br&gt;5.NF.B.3: Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.</td>
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<td>4.NF.A.2: Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $&gt;$, $=$, or $&lt;$, and justify the conclusions, e.g., by using a visual fraction model.</td>
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### Video Clips by Chapter

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Video Clips by Grade, Including Demographics

Demographics: The student body at Lighthouse Community Charter School comprises 81 percent Hispanic, 9 percent African American, 5 percent Multiethnic, 3 percent Asian/Pacific Islander, 1 percent Middle Eastern, and 1 percent Caucasian. Eight-one percent of students are English learners. Eight-six percent of the students receive free or reduced price lunch.

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| Grades 3, 5 | Julie McNamara is the coauthor of *Beyond Pizzas & Pies, Second Edition*, and is thrilled to be a guest teacher in Ms. Hofmayer’s class. She is sure she learned far more from the students than they learned from her! | 5a Reviewing the Names of the Pattern Blocks  
5b Introducing Activity 5.1: Pattern Block Fractions, Version 2  
5c What Do You Call the Triangle When the Trapezoid Is the Whole?  
5d Stephanie’s Use of the Pattern Blocks to Support Her Reasoning  
5e What Do You Call the Rhombus When the Trapezoid Is the Whole?  
5f Is \( \frac{1}{2} \) Always Greater Than \( \frac{1}{3} \)? |
| Grade 3 | Mr. Seay previously taught third and fourth grade and has been teaching for more than ten years. He is currently the K–4 Academic Intervention Specialist at Lighthouse Community Charter School. He enjoys helping students build a strong foundation in mathematics and helping them critique each other’s reasoning. | 2a Introducing Activity 2.1: Number Line Activities with Cuisenaire Rods  
2e Labeling the Number Line  
2g What Do You Know About Numbers Between 0 and 1?  
2h What Are We Learning About Numbers Between 0 and 1?  
2i Revisiting the Question, “What Are the Numbers Between 0 and 1?” |
| Grade 4 | Ms. Lee teaches third and fourth grade. She has been teaching for ten years, and has previously taught first and second grades as well. Ms. Lee places a high priority on student-led discussions and emphasizing multiple ways of problem solving in her teaching. | 2b Splitting the Number Line in Half  
2c The Purple Ones Work!  
2d Lizette’s Strategy for Finding the Rod That Is \( \frac{1}{3} \) of the Unit Interval  
2f Using Knowledge of \( \frac{1}{2} \) to Find \( \frac{1}{5} \)’s |

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| Grade 4 | Ms. Thompson has been teaching kindergarten through fourth grade for sixteen years. She currently teaches third and fourth grade. She is National Board Certified in Early/Middle Childhood Literacy, and is the leader of her school’s third through sixth grade math inquiry group. She believes that encouraging and supporting children to make meaning is the foundation of good instruction in every content area. | 3a Introducing Activity 3.1: Measuring with Cuisenaire Rods  
3b “How Many Brown Rods Long Is the Marker?”  
3c Different Names for the Remainder Length  
3d What Do We Call Four White Rods?  
3e Making Monica’s Thinking Public  
3f Using the Rods to Show How $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$ Are Equivalent |
| Grade 5 | Ms. Kretschmar teaches fifth- and sixth-grade math and science. She has been teaching and learning from her students for seventeen years. She puts a high priority on looking deeply at student work and listening to student thinking to inform instruction. | 8a Introducing Activity 8.1: More or Less Than $\frac{1}{2}$?  
8b Labeling the Number Line  
8c Fractions That Equal $\frac{1}{2}$  
8d Describing the Relationship Between the Numerator and Denominator in Fractions Equal To $\frac{1}{2}$  
8e Reasoning About Fractions Less Than $\frac{1}{2}$  
8f Using Academic Language to Describe the Relationship Between the Numerator and Denominator in Fractions Less Than $\frac{1}{2}$  
8g Reasoning About Fractions Greater Than $\frac{1}{2}$  
8h Reasoning About Fractions That Are Equal To, Less Than, and Greater Than $\frac{1}{2}$ |