VIDEO CLIP

Introduction

Watch the video clip "Introduction." Talk with a friend or colleague; what do you see in the introduction that makes you most excited about *Beyond Pizzas & Pies, Second Edition*?

Why This Resource?

If you ask an upper elementary teacher to name the most challenging mathematics topic for students, chances are you'll hear, "Fractions." Many middle school teachers find they have to review fraction concepts and operations before their students can successfully tackle algebra. We've spoken to countless adults who will unabashedly claim they hate fractions and always have. Sadly, students without a strong understanding of fractions find it difficult to progress very far in mathematics. In fact, the California State University/University of California Mathematics Diagnostic Testing Project Workgroup has found that there is a strong positive correlation between students' understanding of fractions and their overall success in mathematics (Gomez 2009). According to the Final Report of the National Mathematics Advisory Panel (2008), understanding fractions is a "foundation skill essential to success with algebra" (3). The report also states that completion of Algebra II correlates highly with future academic success as well as earning potential.

This book and the accompanying video clips are intended to support you as you help your students develop their fraction sense. Fraction sense implies a deep and flexible understanding of fractions that is not dependent on any one context or type of problem. Fraction sense is tied to common sense: Students with fraction sense can reason about fractions and don't apply rules and procedures blindly; nor do they give nonsensical answers to problems involving fractions. Because we believe that fraction sense is an essential component for students' success with fraction operations, this book and the accompanying video clips focus primarily on helping students build foundational fraction concepts.

Why Is Understanding Fractions So Hard?

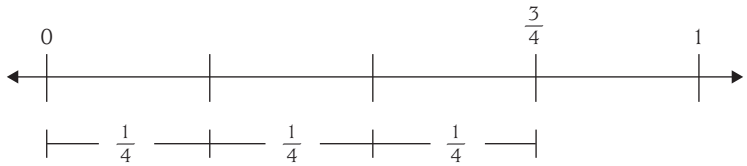
In order to support your goal to help students develop what we call "Fraction Sense," we first need to identify some of the reasons that so many children and adults have such difficulty with fractions.

Helping students develop a deep and flexible understanding of fractions is a complex endeavor. In order for students to understand fractions and fraction notation, they must be able to think about numbers in a different way than when they are working with whole numbers. For example, instead of viewing a number such as 34 as representing a specific quantity, when the same digits (3 and 4) are used in the number $\frac{3}{4}$, the digits 3 and 4 represent a relationship. In addition, students need to consider the context in which the number $\frac{3}{4}$ occurs. While the relationship between the numerator 3 and the denominator 4 doesn't change across contexts, the way the fraction is represented does.

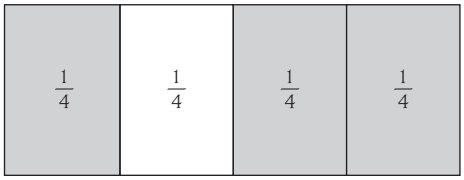
Research identifies several factors that likely contribute to students' difficulties with fractions, including but not limited to:

- The way that fractions are written (Ball 1993; Davydov and Tsvetkovich 1991; Lamon 2007; National Research Council 2001; Smith 2002);
- Classroom practices designed to help students make sense of fraction values and notation that inadvertently mask the meaning of fractions (Armstrong and Larson 1995; Davydov and Tsvetkovich 1991; Lamon 2007; Mack 1990, 1995);
- Students' overreliance on whole number knowledge (Mack 1990; Saxe et al. 2007); and
- The many meanings of fractions, such as measure and ratio (Lamon 2001; National Research Council 2001).

When considering $\frac{3}{4}$ as a number, the 3 represents three one-fourths, and the whole, or unit, is one.

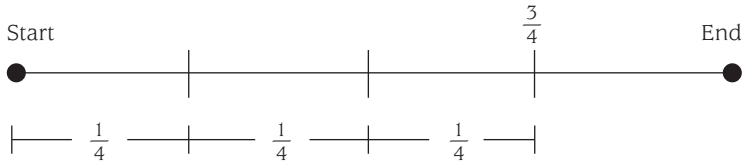


When considering $\frac{3}{4}$ as part of an area, the 3 represents three replications of the area that is one-fourth of the whole.

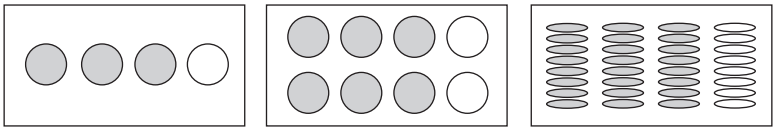


$\frac{3}{4}$ of the rectangle is shaded

When $\frac{3}{4}$ is considered as a measure, the 3 is three iterations of the distance that is one-fourth of the whole.



When $\frac{3}{4}$ is considered as part of a set, the 3 could mean three items, six items, twenty-four items, and so on, depending on the size of the entire set.



How This Resource Is Organized

This book is not necessarily intended to be read from cover to cover. Each chapter discusses one common dilemma that students have with fractions and includes classroom strategies and activities for preventing and addressing these dilemmas. Different chapters may have different levels of significance for you, depending on your grade level and your students' particular needs. The order in which you read the chapters doesn't matter; each chapter is written to stand alone.

Each of the eight chapters follows the same format:

- *CCSS Connections:* Chapters open with connections to the relevant Common Core State Standards.
- *Classroom Scenario:* Each chapter identifies a dilemma that students encounter as they begin their formal study of fractions. Some of these

dilemmas may be tied to fraction notation, some may be connected to students' previous experiences with fractions, and some may be the result of misapplying rules and procedures students learned when working with whole numbers. Each dilemma is presented in the context of a fictional classroom episode in the third-grade classroom of Mr. Burnett, the fourth-grade classroom of Ms. Alvarez, or the fifth-grade classroom of Ms. Chu. While the exact episodes and students are fictional, the students' comments and struggles are taken from our work in classrooms as either teacher or researcher. The dilemmas are also not grade-level specific; you may find that your fifth graders struggle in the same way as Mr. Burnett's third graders or your third graders may have the same difficulties as Ms. Alvarez's fourth graders.

- *What's the Math?* This section is intended to clarify the mathematics that is being addressed in each chapter.
- *What's the Research?* As teachers are asked to do more and more during the school day, it's imperative that we no longer continue with practices because "that's the way we've always done things." The research helps us to identify those strategies, contexts, and representations that may be problematic and/or limited, as well as those that will ensure we get the most out of our instructional time.
- *Classroom Activities:* Each chapter includes activities designed to help your students develop their fraction sense. Materials lists, reproducibles, and examples of student projects are included. *In addition, the accompanying video clips feature actual classroom footage of students and teachers engaged in several of the Classroom Activities.* These activities are not meant to replace your current curriculum, but you may find that they will allow you to provide experiences for your students that help them further develop their fraction sense.
- *Wrapping It Up:* Each chapter ends with closing comments, study questions, and suggestions for additional resources when appropriate.
- *Video Clips:* Chapters 2, 3, 5, and 8 include video clips that were filmed in actual classrooms; see the tables on pages xxxi–xxxiv for a listing of clips by grade and chapter.

What's New to the Second Edition?

The second edition of *Beyond Pizzas & Pies* is updated to reflect the authors' most current thinking and research. In addition, the second edition now includes 30 video clips filmed in actual classrooms. Clips range from 1 minute to 6 minutes in length with a total viewing time of approximately 70 minutes.

The 10 Essential Strategies

Following is an overview of the eight main ideas discussed in each of the chapters plus two additional strategies that permeate throughout the chapters. These two final strategies are not specific to fraction instruction. Instead, they are essential instructional techniques regardless of the content you are teaching. We feel, however, that they are particularly important to consider when helping students develop fraction sense because of the complexity of the topic and the many challenges students and teachers often face when it comes to the learning and teaching of fractions. We hope that you will find these to be valuable additions to your repertoire of teaching strategies.

Strategy #1: Provide opportunities for students to work with irregularly partitioned, and unpartitioned, areas, lengths, and number lines.

By providing opportunities for students to create partitions and reason about unequally partitioned shapes, you can help them develop a deep and flexible understanding of part–whole relations. As we discuss in Chapter 1, these kinds of experiences can help students move beyond a shallow understanding of part–whole relations that is based on merely counting parts to one that is based on truly understanding the relationship between the shaded part and

Ten Essential Strategies for Supporting Fraction Sense

1. Provide opportunities for students to work with irregularly partitioned, and unpartitioned, areas, lengths, and number lines. (Chapter 1)
2. Provide opportunities for students to investigate, assess, and refine mathematical "rules" and generalizations. (Chapter 2)
3. Provide opportunities for students to recognize equivalent fractions as different ways to name the same quantity. (Chapter 3)
4. Provide opportunities for students to work with changing units. (Chapter 4)
5. Provide opportunities for students to develop their understanding of the importance of context in fraction comparison tasks. (Chapter 5)
6. Provide meaningful opportunities for students to translate between fraction and decimal notation. (Chapter 6)
7. Provide opportunities for students to translate between different fraction representations. (Chapter 7)
8. Provide students with multiple strategies for comparing and reasoning about fractions. (Chapter 8)
9. Provide opportunities for students to engage in mathematical discourse and share and discuss their mathematical ideas, even those that may not be fully formed or completely accurate.
10. Provide opportunities for students to build on their reasoning and sense-making skills about fractions by working with a variety of manipulatives and tools, such as Cuisenaire rods, Pattern Blocks, Fraction Kits, and ordinary items from their lives.

the whole, the distance from 0 in relation to the unit distance on a number line, or the relationship between rods of different lengths.

Strategy #2: Provide opportunities for students to investigate, assess, and refine mathematical "rules" and generalizations.

It is not uncommon for children to misapply generalizations as they attempt to make sense of new and complex material. Helping them question and refine generalizations and strategies is extremely important in supporting students' development as mathematical sense-makers. In Chapter 2, we describe some activities you can use in your classroom.

Strategy #3: Provide opportunities for students to recognize equivalent fractions as different ways to name the same quantity.

Students should understand that equivalent fractions—such as $\frac{8}{12}$, $\frac{4}{6}$, and $\frac{2}{3}$—represent precisely the same point on the number line, and the differences in notation (how the fractions are written) are merely a matter of the value of the denominator. As we describe in Chapter 3, students often do not understand that equivalent fractions are multiple ways to name one quantity.

Strategy #4: Provide opportunities for students to work with changing units.

We need to help students understand that an object such as a triangle pattern block can be one-third of one thing (a trapezoid) and one-sixth of another (a hexagon). Activities like the ones we describe Chapter 4, which use materials such as Cuisenaire rods and Pattern Blocks and designate different items as the whole (or unit) can support students' understanding that a fraction is not a name for a given block but a relationship between the block and the whole.

Strategy #5: Provide opportunities for students to develop their understanding of the importance of context in fraction comparison tasks.

Fractions are representations of quantities, and these quantities are measured in relation to a unit (or a whole). The meaning of $\frac{2}{3}$ is determined in part by the size of the unit. As we describe in Chapter 5, students need opportunities to think about the importance of context in fraction comparison problems.

Strategy #6: Provide meaningful opportunities for students to translate between fraction and decimal notation.

Fraction and decimal notation are two different notational systems for rational numbers. Frequently, students do not see that a fraction and a decimal are merely two different ways to name a quantity. As we describe in Chapter 6, providing students with opportunities to work simultaneously with fraction and decimal notations supports their development of fraction sense.

Strategy #7: Provide opportunities for students to translate between different fraction representations.

In the elementary grades, students are introduced to multiple representations for fractions, including shaded parts of areas, parts of sets, and points on the number line. In Chapter 7 we discuss the importance of asking students to translate between different fraction representations. When students translate, they are forced to consider the features of the representation that are representation-specific, as well as those that have mathematical meaning beyond the representation.

Strategy #8: Provide students with multiple strategies for comparing and reasoning about fractions.

Students often rely on a common denominator strategy for comparing fractions, even when other strategies are more convenient or efficient. As we describe in Chapter 8, you can support your students in deepening their fraction sense by providing opportunities for them to use benchmarks to reason about fraction value, and by focusing their attention on the *relationships between* numerators and denominators (not just numerators *or* denominators).

Strategy #9: Provide opportunities for students to engage in mathematical discourse and share and discuss their mathematical ideas, even those that may not be fully formed or completely accurate.

In addition to providing opportunities for students to learn from their classmates, to refine their thinking by explaining to another person, and to use mathematical language in a meaningful way, classroom discussions can provide you with invaluable insights into what students do and do not understand about a given topic.

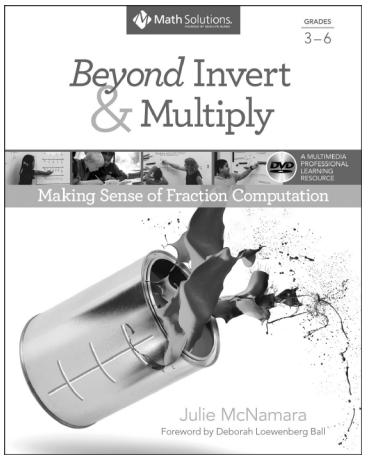
Strategy #10: Provide opportunities for students to build on their reasoning and sense-making skills about fractions by working with a variety of manipulatives and tools, such as Cuisenaire rods, Pattern Blocks, Fraction Kits, and ordinary items from their lives.

The more opportunities that students have to apply their fraction knowledge to solve problems involving different materials, contexts, settings, and relationships, the deeper and more flexible their understanding of fraction concepts will become.

Get Started!

The importance of helping students develop a deep and flexible understanding of foundational fraction concepts, such as those discussed in this book, cannot be overstated. The scenarios and research findings presented in the followings chapters illustrate many of the challenges students without fraction sense face as they attempt to solve fraction problems. By providing opportunities for your students to investigate, discuss, revise, expand, and refine their understanding of fractions, you can prepare them for success with fraction comparison and computation. This preparation will not only help them with fraction tasks they encounter in school, but it will also help them better appreciate and understand the important role fractions play in their world.

See Also



Beyond Invert & Multiply: Making Sense of Fraction Computation, Grades 3–6
A Multimedia Professional Learning Resource

This resource builds on the foundational understandings described in *Beyond Pizzas & Pies* and applies them to situations involving fraction computation. Throughout the resource connections are made to the "Ten Essential Strategies for Supporting Fraction Sense" previously listed.