

A Letter from Math Coach Lori MacDonald

I cannot recall a time in my professional life when I've been more grateful for a resource. Teachers in Berkeley have been working hard learning the Common Core standards, paying special attention to the new emphasis on fractions. The standards seem logical, but we have many questions and some concerns as we enter our second year teaching them. Julie McNamara and Meghan Shaughnessy, in an immeasurably helpful way, have anticipated our situation and designed *Beyond Pizzas & Pies, Second Edition: 10 Essential Strategies for Supporting Fraction Sense* to lead us, step-by-important-step, into greater understanding of the standards and ways we might teach them effectively.

Many teachers are wondering, even after studying the standards carefully, "How am I supposed to teach something I don't yet fully understand?" My answer is, read *Beyond Pizzas & Pies*. Here's a quick look at how each section provides invaluable support.

Classroom Scenario

In the "Classroom Scenario" sections of *Beyond Pizzas & Pies*, the authors get right inside our heads. Through their deft analysis of teachers' thinking process, Julie and Meghan assure the reader that they understand our need to be led through a guided process to gain greater understanding. If we could have mastered the standards in one reading, we would have. Julie and Meghan begin each chapter with us in the classroom as we struggle with knowing how best to teach these unfamiliar concepts.

What's the Math?

The authors then begin a short but effective section of direction instruction, "What's the Math?," and we are listening, because we know Julie and Meghan get it. Having then acquired a better understanding of the concept at hand, we are pushed to consider what the research says. *Where are the typical misunderstandings among students? Among teachers? What has been shown to be most effective in terms of framing the concept?*

Classroom Activities

Then, once our trust has been earned and our understanding increased, right about the time we're thinking, "This is great, but I don't have time to translate this into lessons," *Beyond Pizzas & Pies* gives us "Classroom Activities," replete with reproducibles and video clips demonstrating how to teach the concept.

Wrapping It Up

In "Wrapping It Up," we are asked questions that show us how far we've come and challenge us to set concrete goals for implementing what we've learned.

Throughout the book, the reader feels as if she has a personal math guide. This is not an academic book, simply offering some good ideas on how you might approach teaching fractions. It is a systematic walk through of the Common Core State Standards for Mathematics. In the middle of explaining a new approach taken in the CCSSM, the authors remind us of how previous standards addressed similar concepts. By reminding us of how we previously understood (and probably taught) various concepts, our learning of the new standards is greatly facilitated. No ideas in this book are offered in a vacuum; Julie and Meghan create a context for all they discuss.

What a gift *Beyond Pizzas & Pies* is to teachers. Personal. Practical. Professional. I want every teacher and coach in Berkeley Unified to have a copy of this resource and to wear it out: mark it, bend its spine, and mine it for all its riches and classroom videos.

—Lori MacDonald, Coach, K–5 mathematics
Berkeley Unified School District, California

Summary of Connections with the Common Core State Standards

The following connections are also featured at the beginning of each of the corresponding chapters; they are included here as well for quick reference.

Chapter	CCSS Content Standards: Number and Operations: Fractions (NF); Geometry (G)
1	<p>Grade 3 Number & Operations—Fractions <i>Develop understanding of fractions as numbers.</i></p> <p>3.NF.A.1: Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.</p> <p>3.NF.A.2: Understand a fraction as a number on the number line; represent fractions on a number line diagram.</p> <p>Geometry Reason with shapes and their attributes.</p> <p>3.G.A.2: Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.</p>
2	<p>Grade 4 Number & Operations—Fractions <i>Extend understanding of fraction equivalence and ordering.</i></p> <p>4.NF.A.1: Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.</p> <p>4.NF.A.2: Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $<$, $=$, or $>$, and justify the conclusions, e.g., by using a visual fraction model.</p>

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Chapter	CCSS Content Standards: Number and Operations: Fractions (NF); Geometry (G)
3	<p>Grade 3 Number and Operations—Fractions <i>Develop understanding of fractions as numbers.</i> 3.NF.A.3: Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.</p> <p>Grade 4 Number and Operations—Fractions <i>Extend understanding of fraction equivalence and ordering.</i> 4.NF.A.1: Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. 4.NF.A.2: Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.</p> <p>Grade 5 Number and Operations—Fractions <i>Use equivalent fractions as a strategy to add and subtract fractions.</i> 5.NF.A.1: Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. 5.NF.A.2: Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fractions models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.</p>
4	<p>Grade 3 Number and Operations—Fractions <i>Develop understanding of fractions as numbers.</i> 3.NF.A.1: Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.</p> <p>Geometry <i>Reason with shapes and their attributes.</i> 3.GA.2: Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.</p> <p>Grade 4 Number and Operations—Fractions <i>Extend understanding of fraction equivalence and ordering.</i> 4.NF.1: Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.</p>

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5	<p>Grade 3 Number and Operations—Fractions <i>Develop understanding of fractions as numbers.</i></p> <p>3.NF.A.1: Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.</p> <p>3.NF.A.3: Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.</p> <p>3.NF.A.3.D: Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.</p>
6	<p>Grade 4 Number and Operations—Fractions <i>Understand decimal notation for fractions, and compare decimal fractions.</i></p> <p>4.NF.C.6: Use decimal notation for fractions with denominators 10 or 100.</p> <p>4.NF.C.7: Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of the comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual model.</p> <p>Grade 5 Number and Operations—Fractions <i>Apply and extend previous understandings of multiplication and division.</i></p> <p>5.NF.B.3: Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.</p>

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7	<p>Grade 3 Number and Operations—Fractions <i>Develop understanding of fractions as numbers.</i> 3.NF.A.2: Understand a fraction as a number on the number line; represent fractions on a number line diagram.</p> <p>Grade 4 Number and Operations—Fractions <i>Build fractions from unit fractions.</i> 4.NF.B.3: Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$. 4.NF.B.3.A: Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. 4.NF.B.3.D: Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators.</p> <p>Grade 5 Number and Operations—Fractions <i>Use equivalent fractions as a strategy to add and subtract fractions.</i> 5.NF.A.2: Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators.</p>
8	<p>Grade 3 Number and Operations—Fractions <i>Develop understanding of fractions as numbers.</i> 3.NF.A.3: Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.</p> <p>Grade 4 Number and Operations—Fractions <i>Extend understanding of fraction equivalence and ordering.</i> 4.NF.A.1: Explain why a fraction a/b is equivalent to a fraction $n \times a / n \times b$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. 4.NF.A.2: Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.</p>