

About This Resource

My Story

I remember sitting in a class while in middle school and thinking, “Just another day of sitting here doing exactly what my teachers want me to do.” I was a decent student; I did everything I was told to do but did not put forth the effort I should have in order to excel. It wasn’t until my last year in middle school that I had a sudden realization: I needed to quit doing just OK and start trying to do the best I could. That year I had great caring and knowledgeable teachers who made an impact on my motivation to learn; it was then that I decided I wanted to be a teacher who made a significant difference in my student’s lives.

At the beginning of my teaching career, I had the opportunity to work with students in a very non-traditional setting. I was able to take students from varied backgrounds and help them move forward. In my mind, it did not matter if these students had not memorized their multiplication facts or they did not know how to find common denominators—they were going to learn math. We definitely struggled; to this day I sometimes want to call back some of my students and teach them again; as I’ve worked with additional students I’ve learned so much more about the art of teaching.

Occasionally, I run into former students who tell me, “I became a math teacher because of you!” and I wonder when along my career path I had taught them. Twenty years ago I was a different kind of math teacher. Did they become a teacher like I was when I started teaching or did they realize that a change in teaching methodology was needed to connect more effectively with students? I hope for the latter, and that I inspired them to find their own paths to reaching children.

It wasn’t until I started using the curriculum *Connected Mathematics* that I gained insight into and discovered the importance of my students understanding mathematics and not just learning procedures. Teaching isn’t about finding the perfect worksheet or using manipulatives; it is about helping my students make sense of the mathematics and asking the right questions to help them understand it. I finally started really listening to students and their questions. I remember one student asking, “Why do you tell me to reduce the fraction and then tell me they are equivalent?” I realized that this student was doing just as I had in middle school—doing what the teacher wanted but not necessarily knowing why. However, there was a difference; he unlike I, was brave enough to ask why. Changing the way I taught was not easy; in fact,

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it was the most difficult thing I have ever done. I did not always have answers to all the questions asked by my students; but when I realized that was OK. I became a better teacher. Becoming a learner along with my students helped us build a community of learners. I listened and asked questions as a way of helping my students succeed. I continually asked myself, “How can I help my students make the connections between the mathematics concepts I am trying to teach?”

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Why Focus on Proportionality?

The ability to reason proportionally is at the forefront of middle school mathematics. As middle school students develop the ability to reason proportionally and represent proportional relationships in a variety of ways, they continually encounter opportunities to apply this knowledge across all strands of mathematics. The National Council of Teachers of Mathematics (NCTM) *Principal and Standard for School Mathematics* states, “Facility with proportionality develops through work in many areas of the curriculum, including ratio and proportion, percent, similarity, scaling, linear equations, slope, relative frequency histograms, and probability. The understanding of proportionality should also emerge through problem solving and reasoning, and it is important in connecting mathematical topics and in connecting mathematics and other domains such as science and art.” Teaching proportionality-infused lessons throughout the five content strands—Number and Operations, Algebraic Reasoning, Geometry, Measurement, and Probability and Statistics—helps students make connections among concepts. By including proportionality, proportional relationships and proportional reasoning in all these lessons, we can begin to think of proportionality as the “big idea” that helps us to make this unifying connection across all strands. Teachers can further help students see middle-school mathematics not as separate and unrelated concepts, but as concepts that revolve around a unifying idea or skill.

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Throughout my years of teaching, as we moved from one content strand to another, students often did not see the connection among the topics. Frequently students viewed each new topic as another unrelated or separate concept with no mathematical connection to anything previously studied. Upon reflection, I resolved to use lessons and teaching strategies that made these mathematical connections more evident, so that students would recognize the purpose and value of their prior knowledge, and subsequently use that knowledge as a tool to build a strong foundation in middle school mathematics.

What Kinds of Lessons Are Included?

I have included eighteen lessons that together address the five content strands of middle-school mathematics. These lessons have goals that can typically be found in standards throughout the middle grades. Each lesson incorporates various strategies and the language my students have used when exploring these topics. Ideally we want middle school curriculum to move away from the limiting view of proportionality traditionally promoted—when we “set up the proportion and solve for x ”—to a more inclusive view. This inclusive view encompasses a variety of student-driven strategies, making use of and connecting to students’ prior learning. As students learn new concepts, we want them to use the mathematics they already know to evaluate each new situation. Considering whether a situation is proportional or non-proportional will sometimes help students begin to reason towards a solution. Thinking this way also helps students build a foundation for studying many other topics learned in the middle grades.

How Are the Lessons Formatted?

Each lesson follows a similar format as outlined below.

Correlations

Each lesson is correlated to the Common Core State Standards and the NCTM standards. These correlations are listed in each lesson as well as in quick-reference tables at the front of the resource. Each lesson is also correlated to four of the leading middle school mathematics programs.

Lesson Introduction and Teacher Suggestions

Of significant importance to improving instruction, each lesson features carefully detailed teacher notes. The teacher notes section states the goals for the lesson and establishes the focus for teaching the lesson. An introduction to the lesson provides teachers with suggestions on how to begin the lesson and introduces essential vocabulary to students. The student engagement section offers suggestions for what the teacher should be doing while students are working on the lesson. This section also includes three types of questions to ask students as they progress through the lesson: questions for struggling students, questions for students on task, and questions to extend student thinking (See more details about the question sections below.). The teacher notes end with a summary section that provides suggestions for bringing together any strategies and representations that the students used to make sense of the mathematics in the lesson.

It is important as you prepare to teach the lesson, that you first complete the lesson yourself, and read the teacher notes. This will enhance the understanding of the teaching notes offered within the lesson.

Materials and Vocabulary

Each lesson lists any materials needed to complete the lesson, as well as suggests key vocabulary words. Note that the mathematical vocabulary may vary depending on the placement of the lesson during the course of your curriculum.

Student Engagement

Questions

The student engagement section of the teacher notes includes three types of questions to ask students as they progress through the lesson.

Questioning is a very important component of teaching—it is key to helping teachers assess their students' understanding of the mathematics. I might have the best-planned lesson, but if my students do not truly understand the mathematics, the lesson itself is meaningless. In my earlier teaching years I would delude myself that my students understood the mathematics when in fact they did not. I remember having great lessons and then giving a test only to have my students not do well. This is a painful lesson that I think many young teachers experience. In time I learned I could not afford to wait until I assessed my students formally to find out if they understood the mathematics. I resolved to start assessing daily through questioning. As I became skilled at assessing through questioning, I recognized the need to have primarily three types of questions ready for each lesson I taught: questions for struggling students, questions for students on task, and questions for students to extend their thinking about the mathematics. This resource provides questions in each category to help you begin the questioning process. The questions provided are just a starting point for the many questions you can and should ask students to verify their understanding, get them started, or pique their interest to think deeper about the mathematics concepts.

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- Questions for Struggling Students
- Questions for Students on Task
- Questions to Extend Student Thinking

Questions for Struggling Students

As a teacher, I used to assume, as students began a task, that they knew what they should do to solve the problem. The lack of initial questions from students often gave me the false impression that students would all get started right away on the task at hand; you can understand my surprise when they wouldn't. Sometimes it's a lack of motivation, but sometimes it's a student's uncertainty on how to enter into the task.

When I go up to a student or group of students who have not begun work on the assignment, I don't want to ask them, "Why aren't you working?" Instead I want to ask them questions that they are going to be able to answer in order to begin their thinking process. Often students shy away from my questions because they have not

been successful in mathematics. I want them to feel success and be able to answer the questions I pose to them. Some teachers might say that the questions in this category are too simplistic. These questions are intentionally simple so that all students can answer them and gain motivation in getting started. The questions help the slow-to-begin student feel successful with a challenging concept. My hope is to move students into the next level of questioning as their thinking progresses.

Questions for Students on Task

Frequently we ask questions when we feel students are thinking incorrectly. It follows that students automatically think they are doing something wrong whenever we ask questions. We need to break this mold and ask questions of all students all the time. Through questioning while students are on task, teachers gain insight into a student's thinking and the connections the student is making. The types of questions in this category will help teachers form their summaries for the lesson. How are students solving the problem? What strategies are they using? How can I use their thinking to help other students make sense of the mathematics?

Other questions will come up depending on the student responses. These questions are provided to help teachers start the questioning process. They are not necessarily in sequential order, nor do they all have to be asked of every student. The questions should serve as a guide to foster a climate of always asking questions in class.

Questions to Extend Student Thinking

When students are set to task, whether they are working independently or in groups, they often finish at different times. While others are continuing to work, how do we keep students that have finished engaged? What kinds of questions can I ask students to push their thinking? If a group has finished before others, this indicates that those students are ready to think deeper about the mathematics in the situation. Instead of assigning homework or more practice problems, raise the bar in the current task to press students to delve deeper and make more connections within the mathematical problem. What questions can we ask to encourage students to continue to investigate the mathematics? The questions in this category are meant to have students working beyond the learning expectation of the lesson. Taking the mathematics to another level depends on knowing your students well enough to be able to push their thinking. These questions do not have to be asked of all students. However, we do need to validate the time students spend answering these questions. How do we add this information to the summary and honor their work? The information can be posed as extensions or examples for the rest of the class to think about. Not all students need to fully understand these ideas, rather the ideas can serve as a catalyst for students to think about as they continue to learn mathematics.

In my classroom, asking questions is a two-way street; whenever I ask questions to check for understanding my students may ask a question in response. When students ask questions, we need to make sure their questions get considered, whether through answering with a question or by listening to their peers' thoughts and explanations.

Lesson Summary

The summary section of the lesson often is the most crucial part of the lesson. Teachers need to devote time to planning the summary, yet still have flexibility to modify it when students need additional help. During the summary teachers are encouraging students to make sense of the variety of methods and strategies used. They need to determine how to guide discussions, what needs to be generalized and what connections can be made among the mathematical topics. Teachers need to keep focus on the mathematical goal(s) of the current lesson yet still think about groundwork for future lessons.

It is during the summary that students should exhibit understanding of the topic. As the summary progresses student strategies emerge and teachers perceive how they are interpreting the mathematics. During the summary, students should be asked to share thoughts, question, explain, describe, listen, observe differences and similarities, analyze, reason, revise, make connections, and develop rules and generalizations.

Suggestions are given in the Lesson Summary for accomplishing the elements named above. The goal of the summary is to bring some closure to the mathematics learned that day. Not every question needs to be answered; not everything needs to be written down by students; not every thought heard needs to be correct; not every answer needs to come from the teacher, and not every topic needs to be taught in isolation. A lesson summary is a learning experience that involves the whole class and is facilitated by the teacher to help students understand the mathematics. I think of the summary as the “heart” of the lesson.

Student Recording Page

Each lesson contains student work pages. These pages provide the problems students will solve. The work page starts with an engaging scenario and ends with questions the students need to answer. Teachers should work through the lessons before using them with students, specifically thinking about ways their students might solve the problems.

Student Recording Page Solutions

Each lesson also contains an answer key. The answer keys offer sample solutions and possible strategies students might use to solve the problem. Most of the responses provided in the answer keys stem from actual student responses that I’ve encountered in my classrooms. Teachers should expect students to model a variety of strategies that mirror how they reason proportionally, thus encouraging students to take ownership of their own proportional reasoning and make connections to prior learning.