Part One

Lessons

Scholastic Book of Lists

Fun Facts, Weird Trivia, and Amazing Lists on Nearly Everything You Need to Know!

Scholastic Book of Lists, by James Buckley Jr. and Robert Stremme (2006), contains many facts and trivia, organized by themes. For example, the history section includes lists of presidents, patriotic sayings, and tribes of Native Americans. At the end of each section, there is a survey for students to take that relates back to the facts presented in that section.

The book spans the curricular topics of history, social studies, science, mathematics, languages, and the arts. It also includes lists related to weather, pop culture, sports, and more.

This lesson is about the probability of independent events happening. Students use attribute blocks to explore this concept, then apply what they've learned to figuring out the probability of having both an astrological and a Chinese animal sign in common with one of their peers. Lastly, students find the formula for determining the probability of two independent events.

MATERIALS

attribute blocks, 1 set per small group of students and 1 set for the teacher

grouping circles, made of plastic or string, to create a Venn Diagram

attribute block probabilities record sheets, 1 per small group of students (see Blackline Masters)

overhead transparency of attribute block probabilities record sheet

Introducing the Investigation

I began the lesson by asking students if they knew what their birth sign was. Several students called out their signs. Others were not sure, but using the dates in the *Scholastic Book of Lists*, I helped them figure it out. I asked students what they thought the chance was that someone else in the class had the same birth sign as they did.

I said, "If you think it is likely that someone else has your sign, raise your hand." When students were ready, I called on Wendy and asked her what her sign was.

"Aquarius," she said.

Then I asked who else was an Aquarius. Two other students raised their hands. I did this for two more signs; only one student had the third sign. I explained to the students that we would be looking at the probability that they would have the same astrological signs and Chinese signs as one of their classmates. We figured out that even though there are twelve different Chinese animal signs, each representing certain years, they were all born in only one of three. We discussed that for both of these types of signs, the distribution may not be equal for all possibilities, but that we were going to say they were so we could figure probabilities of equally likely events.

Then I returned to the book and read the topics in the "Grab Bag" section of the book and asked students to select the three they would most like to explore in addition to the first two on astrology and Chinese signs. They picked "gross" food, biggest food, and type of car. I read those lists to students from the "Grab Bag" section of the book (pages 254–73), asking them to write down the foods or cars that made the lists, so that they could pick their choice from among those listed. The entire class was engaged in listening to the lists, groaning at some of the car choices, questioning the gross food options, and expressing awe at how large some of the big foods were. In fact, this introduction lasted more than ten minutes, as students had much to share. (Time could be saved here by selecting only one extra list, as the data for all of these are not needed for the lesson.)

I asked students to look at their choices and to think about how likely it was that someone else in the class had the same answers as they did for each of the five categories. They did not think that this was likely.

I said, "What about having the first two in common—your astrological sign and your Chinese animal sign? That is what we are going to be able to answer at the end of class."

Because students had not explored independent events, I decided to use attribute blocks—a physical model—to illustrate the likelihood of having one trait versus two traits within the set (see Quinn 2001 for more on using attributes to develop a conceptual understanding of probability). First, students had to become familiar with the attribute blocks.

To do this, I set a Venn diagram on the floor using plastic grouping circles (string can also be used). Each circle is about the size of a hula hoop. I labeled one circle *Big* and the other one *Yellow*. I held up two attribute blocks that were different from each other and asked students to tell me four ways we could describe the shapes.

"Color," Lisa said.

"Shape," Cassandra added.

"How about size?" James asked.

I asked, "What are the size choices?"

"Big and small," James replied.

"There is one more thing. Can you see it?" I asked, holding the shapes so that the students could see the thickness.

"Thick or thin," several students replied.

I held up a big circular block and asked where it would go in the Venn diagram. I allowed students time to look at the Venn diagram and then raise their hands. "Whisper to your neighbor where it goes," I instructed. Students agreed it went in the *Big* section of the diagram, but not in the overlap. We continued with five more blocks, deciding where each belonged.

I asked, "If we were to place all of the attribute blocks in our Venn diagram, how many would end up in the *Big* circle? Talk to your partners." After some discussion, students reported back that they thought half would go in that circle because half of the shapes were big and half were small. I asked the same question for yellow, and since the blocks came in three colors, this question was not as easy for the class to answer.

"I think it would be twenty percent because there are less yellow," offered Chris.

"I think ten out of ten because I think they are all yellow," Amy said.

"No, it's one-third because there are three colors, so one in three is yellow," argued Margot.

"Oh, yeah, I was just thinking of the ones in the circle," Amy said. "Of the whole set, there would be only one in three that is yellow, not ten."

I gave instructions to the students for their small-group work. I explained that for each of the possible attributes we had talked about, they were to find the probability of picking a block to match that specific attribute. "For example," I continued, "if you randomly picked a piece, what would the probability be that it was small or that it was red?" I asked one person from each group to get an attribute set and told students to use the blocks to help them find the probabilities.

Before finding these probabilities, I asked students to spend two minutes looking at their blocks and to watch for my hand to go up as a cue that I was ready to ask questions about the blocks. I directed them to see if they could come up with a combination of size, thickness, color, and shape for which no piece existed.

When students got their blocks, they began to sort them by a characteristic. Some groups found all the shapes and then made stacks by color. Other groups created arrays, where the rows were all one shape and the columns were organized by shape, size, and then thickness. Several groups noticed that they were missing a shape or had an extra block. I stopped the groups to clarify that they should each have sixty shapes, none of the blocks should be the same as another block, and they should have five shapes, three colors, two thicknesses, and two sizes. Once they had finished sorting, students figured out that there was exactly one shape for each combination of four traits; for example, there is exactly one big, red, thick square.

I distributed one Attribute Block Probabilities record sheet to each group and placed the transparency of the record sheet on the overhead. We briefly discussed the notation for probability; for example, writing P(red) to mean *What is the probability of red?* I asked each student to record his group's findings in his own notebook as well as recording together on the group record sheet.

Observing the Students

Because of our discussion prior to using the attribute blocks, the students had no trouble identifying that there were twenty red shapes. Many groups figured this out by counting, not by knowing that twenty is one-third of sixty, a strategy that some students did use. Most students did take this shortcut with the attribute *thick,* as they knew that half the blocks were thick and that half of sixty is thirty.

In one group, students were beginning to work on P(red and thick). Nathan added the following two fractions:

$$\frac{20}{60} + \frac{30}{60} = \frac{50}{60}$$

Emily said, "Fifty over sixty; that seems like a lot."

Nathan replied, "Well, I just added the red and the thick; red and thick equals fifty over sixty."

Emily countered, "But there aren't fifty pieces that are red and thick."

During this discussion, Andre was sorting the pieces, finding all the blocks that were red and thick. So far, he had found ten. "There are ten," he stated.

"OK, so ten," Nathan said and recorded that in his notebook.

I interjected, "Why is it not fifty?"

I got puzzled looks, so I restated my question. "If you look at the process Nathan was using, adding thirty and twenty, why doesn't that turn out to be the number of pieces that are thick and red?"

"Because you can't add them together," Nathan offered in a questioning manner.

Emily added, "It's because you aren't just putting together red ones with thick ones because some are the same."

"So, some are the same?" I probed. "Which ones are the same?"

They all studied their pieces. "The ones that are red and thick are the same."

"Oh," said Andre, "adding gets the shapes that are red but includes thin ones, and the thick is also counting all the colors, so you can't just add them."

I could see they were making progress and left them after saying, "Keep thinking about how you can use the first two answers to help you find the third . . . good thinking here."

In another group, Terence, Charles, and Karyn had found all three answers for the first two investigations, each time by just finding the pieces and counting them. They were moving on to the third investigation, in which they would invent their own combinations. I asked them to pause and look at the work they had done, then tell me how they had found the probability of a block sharing two traits.

Terence said, "We just made piles and counted."

I said, "Let's see if you can be clever and avoid some counting. Look back at the second example. How could you just know how many are thin?"

"Well, we did know on that one because thirty is half of sixty and so there were thirty thin."

"And could you use that strategy for square?"

After a pause, Charles said, "Well, there are five shapes."

"So we can divide sixty by five?" Karyn asked.

"Oh, yeah, sixty divided by five is twelve and there were twelve squares," Charles said with confidence.

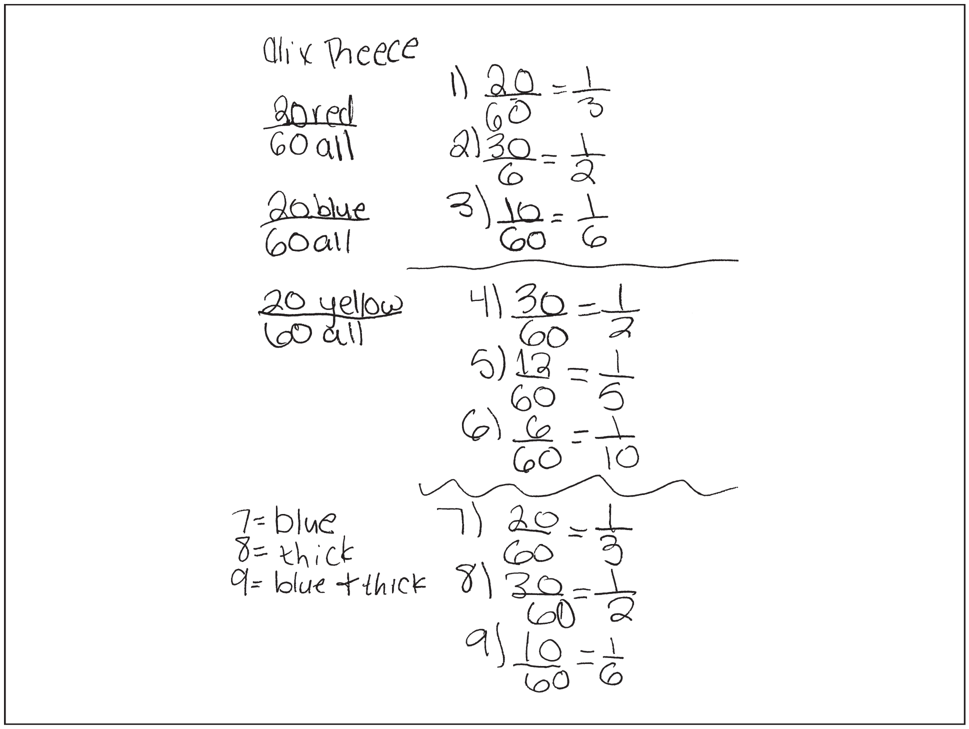
"What about your simplified fractions? How can you tie those in to your thinking about shortcuts?" I asked.

"Our fraction for squares equals one-fifth . . . oh, there are five shapes, so one in five," said Terence, still pondering if this was a coincidence or not.

"Good thinking here," I said. "Keep looking for those clever ways to figure out the probability, because when we share I will be asking for strategies for finding the probability of two traits."

A Whole-Class Discussion

After about twenty minutes, most groups were completely done, but some had not gotten to the third investigation, which I had actually designed as an extra challenge for the quicker groups. (See Figures 1–1 and 1–2.)


Alix Theece

20 red / 60 all

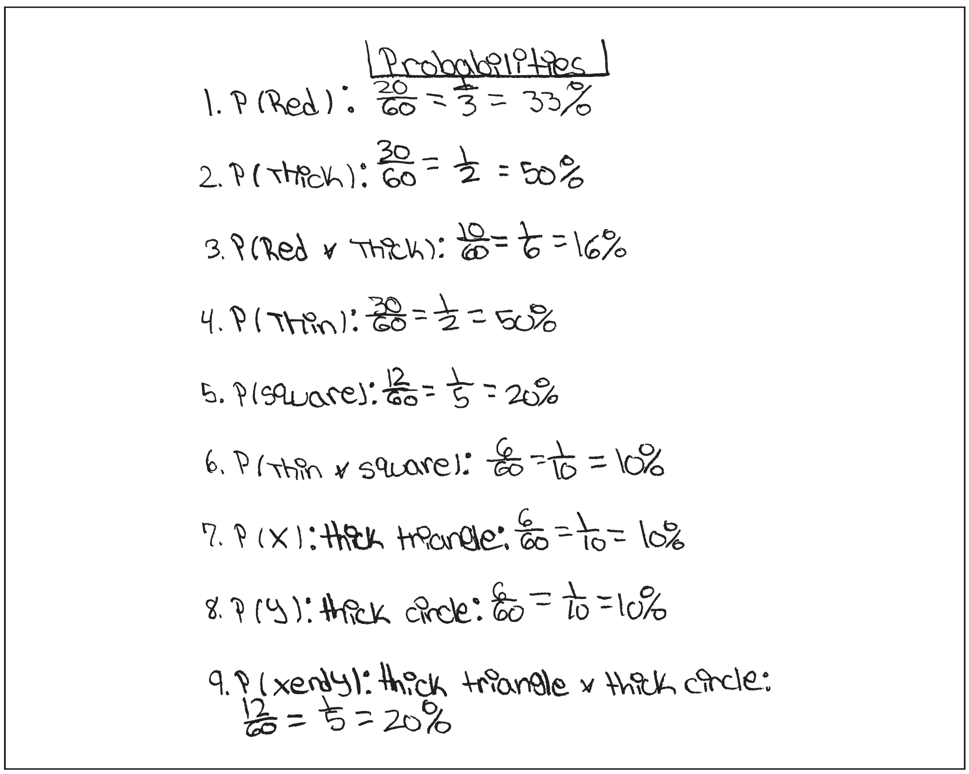
20 blue / 60 all

20 yellow / 60 all

7= blue
8= thick
9= blue + thick

1) 20/60 = 1/3
2) 30/6 = 1/2
3) 10/60 = 1/6
4) 30/60 = 1/2
5) 12/60 = 1/5
6) 6/60 = 1/10
7) 20/60 = 1/3
8) 30/60 = 1/2
9) 10/60 = 1/6

Figure 1–1: This group finished all three investigations.


Probabilities

1. P(Red): 20/60 = 1/3 = 33%
2. P(Thick): 30/60 = 1/2 = 50%
3. P(Red & Thick): 10/60 = 1/6 = 16%
4. P(Thin): 30/60 = 1/2 = 50%
5. P(square): 12/60 = 1/5 = 20%
6. P(thin & square): 6/60 = 1/10 = 10%
7. P(X): thick triangle: 6/60 = 1/10 = 10%
8. P(Y): thick circle: 6/60 = 1/10 = 10%
9. P(xendy): thick triangle & thick circle: 12/60 = 1/5 = 20%

Figure 1–2: This group expressed the probabilities using both fractions and percents.

I asked students to put their attribute blocks away and turn their chairs to face the front of the room. For each of the nine categories on their record sheet, I asked students to tell me what they had recorded as their answers, both the fraction with a denominator of sixty and the simplified fraction. I asked students to talk about patterns they noticed in finding the probability of two attributes. Several students raised their hands.

"If you just multiply the denominators, you get the probability of the two," Deanna stated.

"Can you explain what you mean on the first set?" I asked, seeing that this was not clear to others in the class.

"Well, the probability of red was one-third and the probability of thick was one-half, so multiply the three by the two and you get six. The probability of red and thick was one-sixth. That is our pattern."

"What do the rest of you think about this?" I asked, leading them to the real question I wanted to get to.

"Yes, it works," said Samantha. "I checked the second example, too."

"Why does this work?" I asked. The class was silent for some time.

"Let's see," I said, "if we take the first example, there is a one-in-three chance it is red." I picked up a red block. "What are my chances of picking red?"

"One-third," some students responded.

"OK, so in my set of red, how many of those pieces are thick?" I asked.

"Half," many of the students said.

I could see this discussion wasn't getting where it needed to. I referred the students back to the unsimplified fractions and asked, "How does the fraction for red compare with the fraction for red and thick?"

"Oh, I get it," said Reta. "It's half as much because half of the red are thick. So twenty out of sixty becomes ten out of sixty."

"So for the square, twelve-sixtieths of the shapes are square, but only half of those are thin, so only six-sixtieths are thin and square, right?" Samantha asked.

I could see that students could use the shortcut, but very few were getting the conceptual connection. I asked students to record the patterns that they had found, knowing we would return to this idea again. (See Figures 1–3 through 1–6.)

> 10. If you have 2 things Red & thick it is less likely that there is going to be a match but there is going to be at least on match. !! :)

Figure 1–3: This student explained an important concept, that there is a lower probability when looking for two attributes.

> 10. if you have 2 thing red and thick we took all the red and thick 1s out and counted the we found the lowest fraction then the percent.

Figure 1–4: This student found all the pieces that fit the attributes and then found the fraction and percent.

Figure 1–5: This student explained a pattern in probabilities that looked like multiplication of fractions.

One pattern that I noticed is $\frac{1}{3}$ and $\frac{1}{2}$ = $\frac{1}{6}$ I noticed that if you multiply $\frac{1}{3} \times \frac{1}{2}$ then it will equal $\frac{1}{6}$. The other pattern that I noticed is $\frac{1}{2}$ and $\frac{1}{5}$ = $\frac{1}{10}$ I noticed that if you multiply $\frac{1}{2} \times \frac{1}{5}$ then it will equal $\frac{1}{10}$.

The way that you can do Red and Thick is if you count all of your Red ones and Thick ones together which is 10 then add all of the pieces together which is 60 and then if you simplify it then it will be $\frac{1}{6}$.

Figure 1–6: This student found all red and then found the thick red to figure out the probability.

To find the probability of getting red and thick is to get all the red shape. If you get thick ones and you have all red and thick $10/60 = 1/6$.

Applying Their Findings

I wanted to see if the students could connect what they had done so far to the prompt at the start of class, so I asked them to go back to their data from the lists in the book. "What is the probability of someone being a Taurus?" I asked.

"One in twelve," they replied.

I asked, "Is it likely we have more than one of those in the class?" Students nodded and said yes. "Raise your hand if you are a Taurus," I directed. Two students raised their hands.

Next I said, "Since you were all born within the same three years, what is the probability of having the same Chinese animal sign?" We were assuming the spread was even, although it wasn't.

"One-third," Melinda said.

I recorded these two probabilities on the board:

$$\text{P}(\textit{astrology sign}) = \frac{1}{12}$$

$$\text{P}(\textit{Chinese animal sign}) = \frac{1}{3}$$

I instructed students to talk to their partners about how likely it would be to have two students in our class with a match for both—for example, if two students were both a Taurus and a Dog. Students discussed this in pairs for several minutes. Several groups made the connection to the strategy they used during the investigations with attribute blocks.

"We think it is one–thirty-sixth because we multiplied the denominators," said Lawrence.

"Yes, we did the same thing—multiplied them," Samuel added.

"Why is this the case? Who can tell me why it makes sense to multiply the denominators?" I asked.

"It's because there are only twelve possibilities for your sign, but then only some of those have the Chinese animal, only one in three," Samantha articulated.

I asked students, "Is it likely that someone else in here has the same two signs that you have?" They had many ideas to offer; most said it was not likely. I asked one student for her signs and asked if anyone else had both of them. No one did. On our fourth try, we did find a match. Everyone wanted to know if he or she had a match, but we had to bring the lesson to a close.

This lesson was excellent in allowing students to make the connections between physical models and the concept of the probability of two events happening at the same time. They learned that with more events, the likelihood goes down. Students were heavily reliant on the models to think through the probability of two events. Some students made the connection from the models to the mathematics of multiplying the probability of each independent event; however, more experiences were needed for most of the class to make this connection.