Introduction

Measurement is one of the very earliest forms of mathematics. For centuries people have measured quantities to cook, to build, to make clothing, to divide land, and to keep track of time and distance. In recent years, our abilities to measure have expanded dramatically; we now measure in order to travel through space, to fabricate molecular-size devices, and to create global positioning systems.

Certainly children use measurement in their daily lives, too, as they compare heights, see how far they can run and jump, keep track of how many days until their birthdays, compare their ages, and celebrate each time they need the next shoe size. As children grow older, they may become interested in sports statistics or world records involving measurement; they may use measurement to rearrange furniture or their rooms or to build items such as birdhouses or model rockets. Clearly measurement must be an important part of the mathematics curriculum, as it helps students make mathematical sense of their lives and prepares them for their future.

Unfortunately, it is easy for teachers to become overwhelmed with the abundance of measurement objectives they are asked to address; teachers often resort to dealing with those objectives by telling students what to memorize. For students who experience this kind of incomplete instruction, measurement becomes a list of terms, numbers, and facts that they easily forget.

We too have encountered the complexities of teaching measurement in our classrooms. We know that it is very easy to become overwhelmed by all that is expected of us. But we also have experienced the joys of teaching measurement in ways that help our students make sense of the mathematics they are learning. We've seen our students come away from these lessons excited about their new and deeper understandings of measurement. We've watched our students develop confidence in their ability to use measurement to understand their world rather than struggle to simply memorize rote formulas. These are the kinds of lessons we want to share with you here.

In this three-book series, Sizing Up Measurement, we have worked to create lessons that focus on essential measurement concepts that are connected to problem-solving contexts. The lessons focus on helping students

- identify the attribute to be measured (for example, length or weight);

- know what it means to measure—comparing the attribute of the item or situation with a unit with the same attribute: lengths must be compared with units of length, areas with units of area, and so on;
- develop an understanding of what it means to measure using standard and non-standard units;
- select a system of measurement to be used—customary or metric;
- understand how benchmark units—such as *a centimeter is about the width of a pencil*—help determine the magnitude of specific units;
- estimate the result of the measurement, both before and after the act of measuring;
- select a measurement tool to assign a number value and determine how accurate they need to be; and
- keep track of results in an organized and useful way.

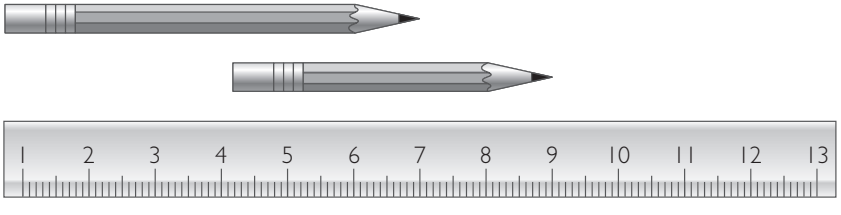
As you can imagine, given the grade-level spans in this series (K–2, 3–5, 6–8), the three books deal with very different levels of mathematics, but there are commonalities among them all. Each of the books includes lessons that relate to categories of measurement important for that grade-level span, and the lessons in all three books provide meaningful contexts for students to solve problems and use their mathematical skills as they develop important vocabulary related to measurement.

Before trying these lessons, it is important to consider the natural progressions in thinking that children pass through as they develop basic concepts of measurement:

When your student lays down toothpicks to measure length and leaves gaps or overlaps the toothpicks, the student is struggling with *unit iteration*. He doesn't yet understand that the distance of the units altogether should be equal to the distance being measured.

When your student thinks that, when measuring with small units, a small total should result, the student does not yet know the *inverse relationship* between the size of the unit and the number of units—small units create a larger total and large units create a smaller total.

When your student compares the length of pencils that are not evenly lined up and thinks that the pencil that sticks out is longer, the student has not yet developed *conservation of length*—the idea that a different position does not change the length.



When your student knows that the marker is shorter than the pencil, and the pencil is shorter than the stick, but doesn't realize the marker therefore must be shorter than the stick, the student has yet to develop *transitive reasoning*. This is necessary in order for children to understand how rulers help us compare objects that are not side by side.

When you put a pencil against the ruler between 2 and 8, and a student thinks the pencil is 8 inches long, the student doesn't understand that the number on the ruler represents the entire distance from the "zero end" to that number.

When a student thinks an angle with longer sides has a larger measure, the student doesn't understand that the measure of an angle depends upon the spread of the angle's rays.

When a student thinks it is impossible to determine the area of an irregular polygon, the student may not understand that figures can be partitioned into shapes that have areas that she can determine.

When a student assumes that a constant perimeter always yields a constant area, the student does not understand the relationship between these two measures.

When a student depends upon a separate formula for determining the volume of each kind of prism and pyramid, the student does not understand the relationships among the volumes of such figures.

The lessons in these books are intended to provide students with opportunities to make sense of these and other critical understandings related to measurement. Through multiple experiences with length, area, capacity and volume, mass and weight, temperature, and time, students learn how to measure, compare, and order. Measurement requires estimation, making comparisons, mental math, and number sense. Students need to add, subtract, multiply, divide, and perceive numerical relationships in many different ways. Measurement is a topic that deserves attention and time in every school year. We offer these lessons in the hopes that you will use and adapt them to fit your circumstances. All students need many opportunities to build their understandings, make connections to other topics, explain their thinking and procedures, and analyze and communicate their results to others. We sincerely hope that you and your students enjoy these lessons.

VICKI BACHMAN, GRADES K–2
CHRIS CONFER, GRADES 3–5
ANN LAWRENCE AND CHARLIE HENNESSY, GRADES 6–8