Introduction

In learning mathematics, second graders regularly demonstrate fragile number sense. As classroom teachers, we witness their developing understandings and are curious about their misconceptions. The following stories will no doubt sound familiar.

At regular intervals throughout the school year, a class of twenty-five second-grade students first estimates, then counts, the total number of pockets on students' clothing on a particular day. Their first pocket-counting experience is in the fall. It is a cool day and many students are wearing pants with pockets. The teacher introduces the activity, then asks, "I want you to look around the room at what your classmates are wearing. Think about the number of children in our classroom, as well as any other important information you could use. How many total pockets do you think students in our classroom are wearing today?" The first student responds with great confidence: "Nine!"

Students in another second-grade classroom keep track of the total number of days that school has been in session, by numbering the days consecutively and working daily with the new number. On the first day of school, the teacher records a *1* on a number chart, and students are asked to collaboratively generate a list of equations equal to "today's number," then "build" that number with various mathematics manipulatives. On the thirty-first day of school, a child yells out, "It's Halloween!"

A group of second graders is introduced to the game *Guess My Number.* The teacher explains the goal of the game: to guess the number he has "hidden" in his head in the fewest number of guesses. He tells students the range of numbers they can consider, writing this range on the board: *1–20*. He asks students to raise their hands to guess possible numbers. He will let them know if the secret number is greater or less than the number guessed and record this information on the board as well. The first student called on says, "I know! Thirty-five!"

Midway through the school year, second-grade students at Lincoln Elementary take their annual trip to the local fire station. They learn about fire prevention, home-evacuation procedures, and what to do in case of fire emergencies. They receive plastic fire helmets, book covers, stickers, and pencils. After practicing how to "Stop, drop, and roll," a recommended action in fire emergencies, they tour the fire station and are invited to hop aboard the ladder truck. The 100-foot-long ladder—clearly labeled with a tag stating *100 feet*—is extended for all to marvel at. One child says to another, "Wow . . . that thing is *really* long! How long do you think it is?" Her friend responds, "Long! At *least* ninety inches!"

The above anecdotes suggest that it is sometimes easier to identify what number sense *isn't* than to identify what it *is*. As teachers of small children, we all share similar stories about our students' struggles with number. We also share the difficult task of addressing their misconceptions and inconsistencies with appropriate, meaningful, and engaging experiences.

What Is "Number Sense"?

This book contains lessons that address aspects of number sense important to consider when teaching second-grade students.

Counting and Estimation

Counting experiences help students to understand that numbers represent quantity. Place-value understanding depends on counting experiences. It is through counting and grouping objects that students come to know the relationship between the placement of a digit and groups of things.

Estimation experiences enable students to consider reasonableness and appropriateness while also creating situations in which students have a stake in the outcome.

Composing and Decomposing Numbers

Number sense includes understanding how numbers relate to each other as well as knowing that numbers can be broken into meaningful chunks. Without explicit experiences that focus on separating quantities into two or more parts, students continue to count to determine quantity. When students have experiences in composing and decomposing numbers, they learn that numbers can be broken apart and put back together in lots of different ways. They develop and use benchmark numbers and show flexibility with numbers and number relationships. The realization that numbers have component parts—including but going beyond groupings of ones, tens, hundreds, and so on—is important for students to develop if they are to operate on numbers in efficient and meaningful ways.

Numbers in Our World

While adults encounter number and computation daily, it can be difficult to convince young children that numbers have purpose and usefulness outside of the four walls of the classroom. Numbers are necessary to locate, quantify, label, and measure. As adults, we are required to interpret and use numbers in a variety of real-world contexts. To prepare them for the world, it is important for our students to become aware of how numbers are used in various contexts as well.

Developing Computation Strategies

The types of numbers we choose, the problems we pose, and the procedures we value have an effect on students' developing number sense. Different people reason in different ways. Rather than memorizing a teacher-demonstrated procedure for computing with numbers, students with number sense are able to understand the procedures they construct and use. They understand and use relationships among operations, as well as knowledge of the base ten system, in order to solve problems. They can explain the procedures they use to others.

Place Value

Place-value understanding grows from experience with counting. When students have frequent opportunities to count objects in many different ways, they come to realize the connections between quantity and the position of a digit. Students that have place-value understanding know that the 6 in 63 refers to sixty, as well as six groups of ten. Until students understand place value, they solve problems based on counting and "one more than" relationships. Understanding that each digit in a multidigit number represents a quantity as well as groups of a specific size is critical for students if they are to solve problems efficiently and flexibly.

Computational Fluency

Students with number sense can transfer their understandings to new situations. They know which methods are appropriate in various problem-solving situations and become more efficient with the methods they choose and use. If students are to develop computational fluency, they need experience with calculating mentally, with paper and pencil, and with a calculator. They also benefit and learn from hearing other students explain their solution strategies.

The Structure of the Lessons

The lessons in this book differ in a variety of ways. Some lessons last one day and others continue over three or more days. Some are one-shot activities, while others are intended to be repeated at regular junctures throughout the school year. To assist you with planning and implementing the lessons in this book, they are organized in the following ways:

Overview This is a brief description of the lesson, and includes the mathematical focus of the lesson.

Materials This section lists the special materials used in the lesson, as well as their quantities. Regular classroom supplies, such as paper, pencils, and scissors, are not listed. Overheads and worksheets are provided in the Blackline Masters section at the back of the book.

Time The estimated length of the lesson, as well as the suggested number of times to repeat the lesson, is included here.

Teaching Directions Directions for the lesson are given in a step-by-step lesson plan.

Teaching Notes Information about the mathematics underlying the lesson is given in this section. It also contains information regarding the type of experiences or prior knowledge students need in order to be most successful with the lesson.

The Lesson This is a vignette describing what happened when the lesson was taught over a period of one to three days. It follows the sequence of the teaching directions, but contains details that are important to consider when planning for the lesson. It also includes samples of student work.

Linking Assessment and Instruction This section contains questions to consider when observing students work. These questions help uncover the degree to which students are connecting with the mathematical focus of the lesson.

How to Use This Book

The seventeen lessons in this book are grouped into sections. The sections are organized by those features of number sense most central to the lessons included, even though these lessons easily share multiple aspects of number sense.

While estimated length and suggested number of class periods are provided for each lesson, it is appropriate that many of these lessons be repeated throughout the school year. This repetition is especially beneficial with respect to lessons that involve playing games. It takes several class periods for students to feel comfortable with the rules for and steps in playing a game. Once students become experienced with these issues, they can begin to focus on the mathematics concepts inherent in the game. Thus, it is beneficial to play games many times, as the purpose for playing them evolves.

Teaching the lessons in this book requires thirty-five to fifty instructional days. However, it is not recommended that these lessons be taught in a continuous manner for seven to ten weeks. After exposure to the experiences and content in a particular lesson, students benefit from time between lessons to reflect upon and take in presented concepts. Lessons within each section are ordered by level of difficulty. It is suggested that lessons that come first in a section are taught earlier in the school year and those that occur last are taught later.

Having students communicate about what they know and can do is an important aspect of this book. Students are asked to regularly explain their thinking aloud and in writing. These explanations serve a number of purposes, important to students and teachers alike. They provide the teacher with valuable assessment information about current conceptual understanding, helping teachers to judge the effectiveness of recent instruction and plan for future instruction. Students deepen their understanding when they explain their thinking to others. Listening to the thinking of others allows students insight into ways of thinking they may not have considered on their own.

Requiring students to explain their thinking through discussion and in writing is not a trivial task. Students benefit from explicit classroom time devoted to modeling effective communication. These lessons provide examples in which students demonstrate what they know and can do both verbally and in writing. Classroom discussions initially focus on the value of listening to and learning from their classmates, as well as helping students recognize the types of behaviors associated with good listening. Students have many opportunities to engage in meaningful talk, in pairs, small groups, and whole-class discussions. Students learn that their thinking processes are valued, and that their contributions are expected, honored, and respected.

Representation is another way that students share their understanding with others. Many lessons detail ways to chart student thinking through the use of words, numbers, pictures, and equations. Recording their ideas not only validates the contributions of individual students but also serves as support: students consider these models as ways to represent their ideas independently.