Lengths of Yarn

CHAPTER 1

OVERVIEW

Measurement experiences provide real-world opportunities for students to use and compare numbers. In this activity, students first estimate the length of pieces of yarn and then determine the measurement using familiar counters as non-standard units. Students discuss the differences between yarn pieces of different lengths, as well as the differences between their estimates and the actual measurements.

MATERIALS

- pieces of thick yarn of different colors, varying in length from 10 to 30 inches, 4–5 pieces per group of 4 students
- Unifix cubes in various colors, approximately 200 per group of students
- *Yarn Lengths* recording sheet, 1 per student (see Blackline Masters)

Yarn Name	Estimate	Measurement	Difference

TIME

- one class period

Teaching Directions

1. Have each student choose a piece of yarn and record its "name" on the recording sheet.
2. Ask the students to estimate the number of cubes that equals the length of the yarn and add this estimate to their recording sheet.
3. Have students determine the actual number of cubes equal to the length of the yarn and add this "measurement" to their recording sheet.
4. Ask the students to compare the difference between their estimate and the actual measurement and add this to their recording sheet.
5. Invite the students to repeat the exercise with another piece(s) of yarn.

Teaching Notes

This measurement activity provides students with real, hands-on, meaningful experience with numbers. Estimating, then measuring, the yarn lengths makes children invested in the result. It also provides an opportunity for students to compare the estimate and the actual measurement. As students continue measuring yarn pieces of varying lengths, they acquire referents to which they can compare successive lengths of yarn. Knowing the number of cubes it will take to equal the length of a piece of yarn makes it possible for students to use this knowledge when estimating and measuring lengths of yarn that are longer and shorter. These repeated experiences allow students to acquire an understanding of how these numbers relate to each other.

The Lesson

Before beginning the lesson, I had cut different lengths of various colors of yarn and attached a masking tape label to the end of each. Each label had a different capital letter written on it, so that students could differentiate one length of yarn from another. I then assembled baskets of materials for each group

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of four students. Inside each basket were four or five lengths of yarn, recording sheets, and cubes.

To begin, I invited the students to join me in a seated circle on the rug. I placed one of the baskets in front of me, then laid out the basket's contents so that all of the children could see the materials.

"What are those?" Daniel asked.

"It's yarn," answered Maria.

"They are all different sizes!" Valerie noticed. "Why do they have tape on the end?"

"There's letters on the tape!" said Todd.

"You noticed many things about the materials in front of me," I acknowledged. "Some of us noticed that these are pieces of yarn and that they are all different sizes, or 'lengths.' Others saw that there is a tape label on the end of each piece of yarn and that this tape label has a letter on it. So we've noticed some important details."

Because the students were going to be estimating and measuring different pieces of yarn, it was important for them to realize that these pieces were indeed of different lengths. I asked them, "If I wanted to know if these pieces really were all different lengths, what could I do to be sure?"

"Lay them all out!" Juan said.

"Put them next to each other . . . then you could see!" Kelly suggested.

"What if you put them side by side? But you have to make the ends all the same," Jonathon offered.

I encouraged him to clarify this idea. "Jonathon, you told me that I would have to make the ends all 'the same.' I am not sure what you mean. Could you tell me in a different way?"

Jonathon got up from his place in the circle and came to the center of the rug. He carefully began ordering all of the pieces of yarn, laying one above the other, matching the left end of each piece so that it was even with the piece below it. "See?" he said. "You can't tell if they're all different unless you make the ends the same." He pointed to the right ends of the yarn pieces. "See how they all stick out different? That means all the yarns are different sizes." The students nodded in agreement.

"Let's find out about their sizes, or 'lengths,'" I said. I pulled out a medium-size piece of yarn from the pile. "See this tape label? This label helps me remember the 'name' of this piece of yarn," I explained. "It helps me remember which piece I have already used." I showed them the recording sheet for the activity. It had four columns, labeled *Yarn Name*, *Estimate*, *Measurement*, and *Difference*. "I am going to write the name of this piece, D, in this column." As I said this I pointed to the *Yarn Name* column and wrote down the letter *D*. "I am going to use cubes to measure the yarn. But first, I wonder how many it would take to go from one end of the yarn to the other?" Many hands went up to assist me with making an estimate.

I called on students one at a time. They gave different estimates, ranging from twelve to twenty-nine. "Mathematicians have a word for these types of 'wonderings,' or 'smart guesses,'" I said. Since the students had been exposed to prior experiences with estimation, I was not surprised to hear them offer, "Estimates!" and "Estimation!" and "Prediction!"

"I am going to do something a little different than 'predict,'" I said. "A prediction is a smart guess about what you think is going to happen. An estimate is a little different. When you make a smart guess about the size of something, or how big it is, that is called an 'estimate.' It isn't a 'wild guess,' because you use what you already know. I have a lot of experience with cubes. I am going to use that experience to make a 'smart guess,' or estimate about how many it will take to equal the length of yarn *D*. I estimate that it will take twenty-nine cubes, snapped together, to equal the length of this yarn." I showed them how to record the estimate in the appropriate column on the recording sheet.

"Now I need to use the snap cubes to measure. I am interested in finding out how close my estimate is to the actual measurement. First, I am trying to remember some important things about using cubes as measuring tools. Can you help remind me of some things to think about?"

Several students raised their hands. "You have to put them next to each other," one student commented.

"No spaces between the cubes," said another.

"They have to go in a line," suggested a third.

"Your suggestions are quite helpful," I said. "I am going to remember them as I measure the yarn. There is something else you will do when measuring the pieces of yarn with cubes. I want you to take one color of cubes at a time and only ten of that color."

"What if you need more than ten cubes?" John asked.

"After you use ten cubes of one color, use ten cubes of a different color. If you still need more cubes, continue to use different colors, each time taking a group of ten. Can someone help me get started with my measurement using this first group of cubes?" I continued.

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When Maria raised her hand, I asked her to come to the center of the rug. "I am going to start with red," Maria said. She counted out ten red cubes and locked them into a continuous "train," placing the train underneath the straightened piece of yarn. "You still need more!" Maria said as she sat back in her place in the circle.

"I wonder how *many* more we need?" I said. Many students whispered their estimates to the students seated near them in the circle. I called on Shannon to assist me. Shannon counted out ten yellow cubes and added them, one at a time, to the train that Maria had begun. She put the completed train of twenty below the yarn. Various students added their ideas. "You *still* need more!" "I think you need five!" "No, she needs more than that!" "I think it will take ten more." The students seemed very interested in finding out how many cubes it would take to equal the length of yarn.

I was interested to see if students would use their place-value understanding to determine the quantity of cubes in front of them, or if they would resort to counting. "How many cubes have we used so far?" I asked.

Some students extended their index fingers toward the trains, pointing to individual cubes and silently counting by ones, while others immediately saw the two groups of ten and raised their hands, saying, "I know! I know!" I asked students to explain how many there were and how they figured out the total. Juan got up from his place on the rug and came to the center of the circle. He counted each cube by ones and said, "There's twenty." After he sat down, Jonathon added, "But, there are two colors of cubes and ten in each group, so it has to be twenty! Ten . . . twenty!" he said, first pointing to the red cubes, then to the yellow.

We still needed more cubes to equal the length of the yarn. Bradley volunteered. He came to the center of the rug, shaking his head and saying, "I don't think it's going to take ten!" He counted out ten green cubes into a pile and added them one by one to the red/yellow train that had already been made. He added six more cubes and said, "I think that's it! I only needed six." He put the remaining four cubes back into the cube pile and took his place in the circle.

"How many cubes did it take to equal the length of the yarn?" I asked. "And how did you figure that out?" Students began pointing at the cubes from their places. Some counted by ones, while other students put their hands up immediately. I gave students the opportunity to again explain how they figured out the total number of cubes. Some of the students explained how they counted the cubes by ones while others reported that they'd counted the two groups of ten and knew that six more would make twenty-six.

"Some of you counted the cubes by ones and some of you counted the groups by tens," I said. "Is there a different way to count the cubes? Carolyn?"

"We could count them by fives!" Carolyn said.

"If we *did* count the cubes by fives," I answered, "would there be more than twenty-six, less than twenty-six, or twenty-six? How many think more than twenty-six?"

Several students raised their hands.

"Less than twenty-six?" I asked. Again, a number of the students' hands went up.

"How many think the number will still be twenty-six?" I asked, and the rest of the students raised their hands.

I asked Carolyn to come to the center of the rug and count the cubes by fives. She picked up the red/yellow/green train and laid it in front of her. She counted five red cubes, snapped off a train of five, counted five more, snapped off the train of five cubes, and continued until she had five trains that were five cubes long and one leftover cube. She counted aloud while touching each train: "Five, ten, fifteen, twenty, twenty-five, twenty-six! I *knew* there would be twenty-six!"

Juan called out, "We could count by twos!" Again, I asked the students if there would be more, less, or the same number of cubes if we counted by twos. This time, fewer students thought there would be a quantity different from twenty-six, but there were still students who thought there would be a different quantity using this new counting method. Juan came up, separating the trains into individual cubes. He gathered the cubes into sets of two and counted aloud, "Two, four, six, eight, ten," all the way to twenty-six. "Yep! There's twenty-six!" he said.

At this point, the students seemed convinced that it did, indeed, take twenty-six cubes to equal the length of the yarn. I showed them where to record *26* on the recording sheet, pointing to the column titled *Measurement*. I pointed to my estimate and said, "You know, I estimated that it would take twenty-nine cubes to equal the length of yarn, but the measurement was twenty-six. Was I close? How close? And how do I know?"

"You were pretty close," Jonathon stated. "You were only . . ." He counted from twenty-six to twenty-nine quietly, extending one finger as he said each number, "three off!"

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When I asked him to explain his thinking, Jonathon touched his forehead and said, "I started with twenty-six in my head, and I counted to twenty-nine." He used his fingers again to keep track: "Twenty-seven, twenty-eight, twenty-nine. It was three more."

"Is there another way to figure out the difference between my estimate and the actual measurement?" I asked.

Carolyn had a suggestion. "You could do it the same way, but go back."

"What do you mean?" I asked.

"You could start with twenty-nine and go to twenty-six, like this: twenty-eight, twenty-seven, twenty-six. It still takes three to go from twenty-nine to twenty-six." Like Jonathon, Carolyn used her fingers to keep track of the difference.

"Jonathon and Carolyn used two different ways to find the difference between my estimate and the actual measurement. Jonathon started with the actual measurement, twenty-six, and counted on to get to my estimate, twenty-nine. Carolyn started with my estimate and counted back to get to the actual measurement. Both used their fingers and both got a difference of three. I am going to record that number in the column labeled *Difference*. This is the last piece of information for your recording sheet before you try a different length of yarn."

The students were getting fidgety; they were anxious to try the activity on their own. Before they did so, I wanted to be sure they were clear about what the activity entailed. "Who can remind us what we should do with our baskets of yarn?" I asked.

"You choose a yarn and write down the letter that's on the tape," one student said.

"You predict—I mean *estimate*—how many cubes long it is. You write that down, too," another offered.

"Then you use the cubes and measure the yarn," another said.

"But don't forget to use different colors!" another stressed. "And you can only take ten of each color!"

"Then you have to figure out how many cubes you were wrong," another explained, "how many cubes between your estimate and the real thing . . . the difference."

"Good! One last thing," I said. "Please count your cubes in at least two different ways." I then had the students go back to their tables, and asked one student from each table to come up front to get a basket containing the lengths of yarn, recording sheets, cubes, and pencils.

As the children began to work I slowly circulated around the classroom, making sure that they were following the proper steps, and also to see how they would approach the task. I was curious about students' counting strategies: would they be able to easily count by twos, fives, and tens? I also wondered how they would find the difference between their estimates and the actual measurement: would they count on from the lesser number or back from the greater number, or would they have sufficient experience with number combinations to "just know" the difference? I was also interested to see if students would build on their earlier findings: would they use their measurement of one piece of yarn to make a reasonable estimate about the length of a subsequent piece of yarn?

Students were eager to get started. Most immediately got a recording sheet, a pencil, and a piece of yarn from the basket. Some students began working as partners. I had initially conceived of this activity as something they would be doing alone, but I realized that by working together students could help each other remember the steps involved and discuss with each other their number choices.

"I think it's going to be twenty-eight. It's almost the same size as the one Ms. Scharton picked," Danielle said.

Her table partner, Aaron, disagreed. "I think it's less. I think it's going to be twenty-four."

Both students wrote down their estimates in the appropriate column of their recording sheets and proceeded to measure with the cubes. Aaron began placing random colored cubes on the desk in a line next to the piece of yarn. This caught the attention of Maria, a tablemate. "Hey, you guys! You have to take all one color!"

"Oops!" Aaron said, and he began removing all of the cubes except the blue ones, gathering more to make a complete set of ten. "You get blue and I'll get green ones," Danielle suggested, and she began bringing together a group of ten green cubes.

When they had connected first the blue and then the green cubes, I asked, "How many cubes have you used so far?" Aaron began touching each cube, one by one, and counting, while Danielle said, "I know!"

"Wait until Aaron is done and see if your idea matches his," I instructed.

Danielle patiently waited until Aaron touched the last cube, stating, "Twenty!"

"I knew it!" Danielle said.

"How did you know?" I inquired.

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Danielle touched the blue cubes and said, "See . . . ten." Then she touched the green cubes and said, "Twenty!"

"So you counted by . . ." I prompted.

"Tens," she said, "and Aaron counted by ones. My way's quicker!"

"That could be," I responded, "but if it doesn't make sense to Aaron, he needs to count in a way he understands. Are you done? Or do you still need more cubes to find out how long the string is?"

Both students looked at me as if I had asked them a very silly question. "Ms. Scharton, we're not done! Look!" and Aaron pointed to a length of the piece of yarn that had no cubes lined up next to it.

"Why don't you think about how many more cubes you'll need to get the right measure," I said, and I left as Danielle and Aaron began to talk about their estimates.

I stopped by Shannon's table. She had just finished measuring her yarn length, and she had a train of five different colors, ten red, ten yellow, ten blue, ten green, and six brown cubes. She had written *38 cubes* as her estimate but she was erasing her number.

"Hi, Shannon. How's it going?"

Shannon looked up at me a bit sheepishly. "Hi, Ms. Scharton."

"It looks like you finished measuring. Good for you! How many cubes equaled the length of your yarn?" I asked.

"Forty-six," she said.

"You know, I should have known that! You wrote *forty-six* right here!" I pointed to the *Measurement* column. She continued to look a little guilty.

"Why are you erasing your estimate?" I asked gently. When she didn't answer, I continued. "You know Shannon, most of the time, when I measure, my estimate is not the same as what I find out by measuring. Remember when I showed the class how to do *Lengths of Yarn*? When I measured my yarn, I didn't get the same answer as my estimate. But you know what? I do my very best job of making a careful estimate. Sometimes I really want to erase my estimate after I do my measuring, but that would be silly. Then it would be a 'matching the numbers' job and not an 'estimating and measuring job.' This activity is an 'estimating and measuring' job. Could you do me a big favor? Could you leave your estimate so I remember what a really careful estimate you made before you measured?"

Shannon rewrote the part of her estimate that she had erased and then began breaking the train into five-cube sections, counting aloud as she snapped them off, "Five, ten . . ."

Kelly and Daniel had just finished measuring a length of yarn they were sharing and were putting their yarn back in the basket.

"You look like you are done measuring," I stated. "Tell me what you noticed."

Kelly began, "We counted by twos and fives and tens and each time we got thirty-two!" Daniel chimed in, "Kelly estimated thirty-six, but I estimated thirty-two! I won!"

"Your estimate *did* match your measuring, Daniel. I bet Kelly put good thinking into her estimate, though. How close is Kelly's estimate to yours?" Daniel looked puzzled, but Kelly started counting aloud, using her fingers: "Thirty-three, thirty-four, thirty-five, thirty-six! Four. Our guesses are four apart!"

"That was fast thinking," I said. "Could you explain how you figured that out?"

"I went 'thirty-two'"—and she put up fingers one by one and said, "'thirty-three, thirty-four, thirty-five, thirty-six.' It took four fingers. It's four!"

"You could also go, thirty-six," Daniel added, putting up fingers one at a time and saying, "thirty-five, thirty-four, thirty-three, thirty-two! It's still four!" He then reached for another yarn piece that was longer than the one he and Kelly had just measured.

"What do you think about that yarn length? Is it longer, shorter, or the same length as the one you just measured?"

"Longer!" they sang out together.

"The last yarn was thirty-six cubes long. How much longer do you think this one is?"

"Oh, Ms. Scharton," said Daniel, "you ask too many questions!"

Linking Assessment and Instruction

As the students work, think about the following:

- Are the students comfortable with "committing" to an estimate? Or do they feel compelled to change their estimate once they have counted the actual number of cubes?
- Do the students become more comfortable estimating as they work through the activity? Does the difference between their estimates and the actual measurements decrease with experience?
- Are they able to count by twos, fives, and tens? How high can they count using each of these ways?

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- Do the students use the information about the number of cubes it takes to equal the length of one piece of yarn to make estimates about the number of cubes it will take to equal the length of another piece of yarn?
- What methods do they use to find the difference between their estimate and the actual number of cubes it takes to equal the length of a piece of yarn? Do they need to build each quantity to compare the difference? Or do they count on or back from one quantity to another? Do they use their knowledge of number combinations to determine the difference between two quantities?
- Do the students realize that the total number of cubes for a particular measurement will not change when the counting method changes?

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