Introduction

Last summer Annika, my five-year-old niece, ran up to me excitedly. "Leo just turned six!" she said. But then she furrowed her brow in thought. "When will I be six?" she asked.

"You'll be six next year," I answered.

"Then we'll *both* be six!" she responded, breaking into a smile.

A few months later I heard Annika talking to Leo. She proudly announced, "When you're seven, I'll be six. When you're eight, I'll be seven. When you're nine, I'll be eight."

In a few short months Annika had developed a new understanding. Her age had a relationship to Leo's: it would always be one less. Furthermore, Annika could use that relationship to predict what her own age would be.

How do children make these shifts in understandings? How do they develop number sense and the relationships that help them describe what they see in the world around them? And what is our role as kindergarten teachers in helping children see numbers not as something mysterious and magical, but as relationships based on logic?

Young Children and Numbers

Young children see math being used in their world, but it takes time for them to understand the purpose and thought behind what they observe. For example, they might see adults point at objects and count, saying, "One, two, three . . ." But they often believe that adults are naming things, in the way that individuals might point at people and say their names: "Fred, Sally, George . . ." Children often do not understand that the final number stands for the entire quantity.

Young children might count five buttons, and when the buttons are spread out more widely, think that the quantity has increased. This is because children trust their visual perception more than their newly developing sense of numbers. They think that by changing the arrangement, the quantity will change as well.

In the same way, children may count five buttons beginning with the green one, but when they count the same buttons beginning with the red one, they think a different number will result. Many young children are not bothered at all when they recount the same set of items and arrive at a different number.

The Importance of Number Experiences

It takes time for children to develop number understandings, and children come to school with widely varying number experiences. Some children have had few, others have had many. For example, Jason's mother consistently talks out loud about her mathematical thinking, and invites him into mathematical discussions.

She'll say, for example, "Jason, how many grapes do you think are in the basket?" As they eat, she'll make other observations. "I see three grapes in this cluster. Do you think there are more grapes in that cluster?" Later she might comment, "Oops! You ate one. Now how many are there?" or, "Uh oh! Your little sister wants a snack, too. You'll need to give half to her so it's fair. How many will each of you get?" Jason participates in conversations like this day after day, month after month, year after year.

Jason might have had an equally loving parent who only knew to say, "Jason, dear, come and eat."

The mathematical concepts and language that children bring to school reflect the mathematical interactions they have had in their lives. Mathematically proficient children have number capabilities not so much because they are "smart" or "gifted" but because their families participate in a multitude of mathematical experiences.

Our job as teachers is to recognize that children do the best that they can. We need to create a mathematically literate environment at school that invites children to think mathematically. We need to invite them into mathematical discussions about what they think. We need to create safe environments that allow them to risk taking those first tentative mathematical steps, in the same way that we create child-safe physical environments in which toddlers can take their first wobbly steps. And just as we celebrate the milestones as our children learn to walk, skip, run, and throw a ball, we must celebrate our students' mathematical milestones whenever they occur.

The Complexity of Number

Numbers are so much a part of what we think about as adults that we have trouble remembering how nebulous they really are. Numbers don't truly exist in and of themselves but are instead an idea. They name an amount in relative terms. Three books is more than two books but fewer than five books. "Three" can be small when we think of three paperclips but is large when we think about three elephants.

Numbers are a way to describe an experience. They help explain the different way I feel when I go outside when it's 70 degrees and I feel the comfortable breeze, breathe in the cool air, and walk happily through the park, compared to when it's 110 degrees and I feel the oppressive heat that constricts my breathing, pushing me to seek shade, hurry inside, or cover my eyes against the intense light.

You can't *see* a number. Three apples may be in front of me. I may see the redness, smell the fruitiness, and eventually taste the flavor and experience the juiciness. But I don't see "three." Three is only an idea or a relationship that we construct.

Number usually tells us how many. We use numbers to decide if we have enough or too little or too much. Numbers arose from humans' need to describe quantity; the first shepherds had to make sure that the same number of

sheep returned as went out into the pasture that morning. Children often need to determine quantity. "It's not fair!" Jonathan protests. "Amanda got more candies than I did."

Numbers have other roles as well. They are used to identify, such as in a Social Security number or a telephone number. Numbers can also be used to measure, or to identify an item in a sequence of other items, such as when we say "the second chair."

What Number Sense Means

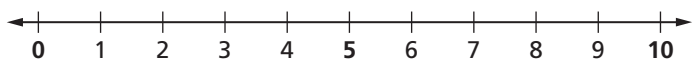
Children with well-developed number sense use numbers to solve problems. They make sense of numerical situations and use what they know to figure out what they don't know. To have good number sense, children must understand the following basic concepts.

Counting

Counting is a complex idea, and foundational to other number concepts. In order to count, children must

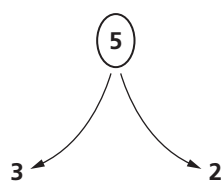
- know the sequence of number names
- touch each item while coordinating the touching with verbal counting (one-to-one correspondence)
- keep track of which items have already been counted
- understand that the last number they say stands for the entire quantity, and includes each of the other numbers they have said (inclusion)
- realize that the items can be rearranged and still be the same quantity (conservation)

Number Relationships



Children need to know how numbers relate to each other—for example, that five is one less than six, but three more than two. This includes comparing "more," "less," "the same," "how many more," "how many less," and the sequencing of numbers.

Decomposing and Composing Numbers



Children need to know that inside a number are other numbers, that they can break numbers apart and see what other numbers are inside. For example, they need to know that inside five is a three and a two. Children need to decompose flexibly, developing the habit of looking for possible combinations of numbers, such as, for five, one and four, and two and three. Young children need many chances to decompose numbers up to five. They will use what they know about these smaller numbers to decompose larger numbers.

Just as children decompose numbers, they need to know that when they recombine those two smaller numbers (such as three and two), the larger number (five) will be the result.

Landmark Numbers

Children get lost in the land of numbers in the same way that I get lost when I visit a new city. Children look for "landmark numbers" to make sure of where they are, just as last week I looked at the mountains around El Paso, Texas, to keep track of where I was.

The landmark numbers that kindergartners often use are five and ten. Children need many experiences with the number five, because everything that they know about five will help them with ten. Later, children will use what they know about ten to help them with larger landmark numbers, such as twenty, fifty, and one hundred. I often encourage this kind of thinking by highlighting landmark numbers on the children's number lines or 1–100 charts.

Strategies for Computation

From all the above understandings, children develop strategies for computation. They use what they know to figure out what they don't know. And over time, children learn to compute quickly and efficiently.

Children develop various strategies for addition, such as:

- Counting all: For the problem 4 + 3, a child might count out a group of four objects and count out another group of three objects. Then the child counts each object starting at one, saying, "One, two, three. Four, five, six, seven."
- Counting on: For the problem 4 + 3, a child might make a group of four objects and another group of three objects, and then touch the group of four and count the rest, saying, "Four—five, six, seven."
- Using doubles: For the problem 4 + 3, a child might say, "I know three plus three is six, and it's one more, so the answer is seven."
- Making fives or tens: For the problem 4 + 3, a child might say, "I'm going to take one from the three and give it to the four, so that I have five plus two. I know that's seven."
- For the problem 9 + 6, a child might say, "I'm going to take one from the six and give it to the nine, so that I have ten plus five. I know that's fifteen."

Strategies for subtraction are related to strategies for addition. But since additive thinking is often easier than subtractive thinking, I spend more time helping young children build number sense through addition.

Children use a wide variety of strategies. I learn from them as they compute in ways that make sense to them. Meanwhile, children come to realize that they know a lot about number relationships. They become more mathematically powerful and learn to trust in themselves as capable mathematicians.

Eventually children "just know" number facts, and that is an important goal. But I want all children to develop strategies for solving problems that they don't automatically remember. Children are challenged by equations such as 25 + 19. They will need ways to figure out any problem, ways that help them use what they know to figure out what they don't know, ways that build number sense.

The Importance of Making Sense of Numbers

We can't simply tell children about numbers and think that they will "know" them. Children will not develop number sense by merely circling answers and writing in workbooks. They have to construct these understandings and build these relationships in their minds, through experiences over time and through discussing with others the relationships they encounter.

Superficial experiences with number may produce a temporary veneer of competence. Children may have the appearance of understanding. But this veneer is likely to crumble in the child's upper elementary or middle school years. Many of us know middle school children who have trouble with fractions, who can't remember procedures for computation, who worry about word problems and say, "Do I add? Do I subtract? Just tell me what to do and I'll do it."

All children deserve the time that they need to construct understandings of number. Children who develop solid relationships with smaller numbers use them as tools for understanding larger numbers. What children know about five helps them understand ten, and then later twenty, fifty, one hundred, one thousand, and then ten thousand. The time invested in the early years to allow children to develop solid number understandings will pay off greatly in later years.

Thoughts About Organizing the Year

While counting is the basis of all number sense, other aspects of numerical understanding do not develop in a linear manner. Children can develop number relationships, decompose numbers, and develop strategies early on. A child who can count to 10 might decompose four, and can compare numbers one through six. As children gain control over larger numbers, they learn to decompose and develop number relationships with larger numbers. However, children's ability to count will always surpass their ability to decompose numbers. When Paula began first grade, she could count to 39, but could easily decompose numbers only up to four. Hannah could count up to 109, but she could decompose numbers only up to five.

When I plan my year's number instruction, I begin by focusing on activities emphasizing counting and comparing number relationships, adjusting the numbers to ranges appropriate for different children. Later I introduce decomposing smaller numbers, again adjusting the numbers. As the year progresses, I spiral back to counting, comparing, and decomposing activities, but increase the magnitude of the numbers as children's abilities to manage numbers grow.

As a teacher, I need to have an idea of where each child is. I try to listen to each child count, compare numbers, and decompose numbers at the beginning of the year and in the middle of the year. This gives me a sense of the range of numbers in which each child needs to work when I'm highlighting specific aspects of number sense.

From the first day of school I also do "Numbers in Our World" activities, including all children in interesting problem solving, even with numbers that may be out of reach for some of them. We can never be sure of what each child knows; children often surprise us. Also, children learn a great deal from one another. Furthermore, all children need to have a chance to participate in the beauty and complexity of mathematics. We don't wait until children master

dribbling and chest passes before we let them play basketball. We read interesting chapter books to children who don't yet read independently at that level. In the same way, all children participate in problem solving, doing real mathematics, learning that mathematics is enjoyable and serves a purpose in their lives.

The Structure of the Lessons

In order to help you with planning and teaching the lessons in this book, each is presented in the same format, with the following sections:

Overview To help you decide if the lesson is appropriate for your students, this is a nutshell description of the mathematical goal of the lesson and what the students will be doing.

Materials This section lists the special materials needed, along with quantities. Not included in the list are regular classroom supplies such as pencils and paper, glue sticks, scissors, and so on. Worksheets that need to be duplicated are included in the Blackline Masters section at the back of the book.

Time Generally the number of class periods is provided, sometimes with a range allowing for different-length periods of time, such as "ten to fifteen minutes." Some activities are meant to be repeated from time to time.

Teaching Directions The directions are presented in a step-by-step lesson plan.

Teaching Notes This section addresses the mathematics underlying the lesson and at times provides information about the prior experiences or knowledge students need.

The Lesson This is a vignette that describes what actually occurred when the lesson was taught to one or more classes. While the vignette mirrors the plan described in the teaching directions, it elaborates with details that are valuable for preparing and teaching the lesson. Samples of student work are included.

Extensions This section is included for some of the lessons and offers follow-up suggestions.

Linking Assessment and Instruction This section offers you some ways to observe children and how you might interpret what you see children do. It relates children's responses to important mathematical understandings, how children represented their thinking, and what strategies they used.

Thoughts About Organizing Instruction

Sometimes I do activities with small groups of children. If the activity is a game, the first group of children I teach can then help teach the others.

Other times I introduce a new investigation, activity, or game in a whole group, with the class, including myself, sitting in a circle on the rug. This way everyone can see the manipulatives I have laid out in front of me. Once the children know how to do the activity, I often use it as part of math workshop time.

During math workshop, children choose from several activities that they already know how to do. I simply place the activities at different tables and children choose what they will do. The number of available chairs at a table

tells the children whether there is room for them to choose that activity. If I want a child to use specific target numbers, I simply write the numbers on a card and hand it to the child.

Math workshop is an important part of my classroom structure because children need to do math activities more than once. The first time, they are just learning how to do the activity and what is expected. The real learning, connections, and language develop more readily when children repeat the activity. Revisiting allows children to focus on the mathematics, refining strategies and trying new, more efficient ones. Furthermore, children enjoy making choices and having control over their learning. And I appreciate being free to sit with an individual child or a small group of children to hear them express their thought processes and to help them.

I frequently have children pair up as "math buddies" during whole-group time. I might have the partners talk to each other or discuss an answer during group discussions. I also have math buddies check with each other when they have a question during work time.

How to Use This Book

This collection of lessons is not intended to be a complete arithmetic curriculum. Instead I envision this book as a guide for kindergarten teachers as they help their students build numerical understanding. It presents a picture of the kinds of investigations, activities, and games that promote development of number sense. It describes the elements that make up the solid foundation in number understandings that children will draw upon as they encounter more complex mathematics in later years.

The first section of the book, "Numbers in Our World," presents lessons that show children how mathematics is important in their lives both at home and at school. These activities help build a classroom culture for learning arithmetic and are to be used throughout the year when mathematical opportunities naturally arise in the classroom. The second section, "Counting," gives children opportunities to count. The third section, "Number Relationships," helps children compare numbers and learn how numbers relate to each other. The fourth section, "Decomposing and Composing Numbers," helps children learn what happens when they break apart numbers and recombine them. The fifth section, "Landmark Numbers," provides some specific activities and models that focus on the numbers five and ten.

I envision this book as the beginning of a conversation rather than a definitive statement of how teachers should teach and what children should learn. Each child is unique, each learning situation different. I hope that teachers will hear my perspective and stories, but will make appropriate adjustments for their students. I hope they'll truly listen to what the children say, try to make sense of it, and change their teaching accordingly. As Marilyn Burns advises teachers, *"Do what makes sense to you . . . and persist until it does."*