Introduction

This book explores effective teaching strategies and mathematical topics that are directed at achieving competence in computation and an enthusiasm for working mathematically in middle and upper primary school students. Reports of teacher and student participation in many lessons involving working with numbers are featured throughout the book.

Working Mathematically

The teaching described in this book targets the development of each child's understandings, knowledge and skills as they work mathematically. 'Working mathematically' children explore mathematical topics and solve problems, developing and applying mathematical techniques as they do so.

Many of the topics and problems in a mathematics classroom can be initiated by the children themselves. In a classroom focused on working mathematically, teachers and children work together as a community of learners; they explore ideas together and share what they find. It is very different to the traditional method of mathematics teaching, which begins with a demonstration by a teacher and continues with children practising what has been demonstrated. The following account illustrates the method of mathematics teaching I have developed and implemented with many classes.

For two years I have teamed with another teacher, Pam Daly, to teach mathematics to middle and upper primary students. Pam and I brought our classes together for a short introduction to each lesson, often drawing on the work of the children to present problems or set a topic for investigation.

The children worked alone or in small groups to produce their lists and search for patterns. When Pam and I joined in with the children as they worked, we monitored each child's progress, noting their understandings,

knowledge and skills, and we intervened to help them develop understandings and challenge misconceptions.

The understandings, knowledge and skills we monitored were not only content based, such as: *How does this child carry out addition? What language does this child use to describe what they are doing when they add?* but were also relevant to how the child worked mathematically: *How does this child begin an investigation? How does this child contribute in a group?*

At the end of each lesson, Pam and I gathered our classes separately in different parts of our shared room. Each class wrote up a list of their mathematical explorations or patterns, drawing on what the class as a whole had done. If my class discovered a new pattern I would go across to Pam's class to tell them. This knowledge sharing is how Pam and I modelled our enthusiasm for mathematics and drew the children into the exciting world of mathematical exploration.

Computation

The children involved in the type of investigation described above were gaining experience in computation as they worked. Problems and investigations are important because, not only do they develop children's ability to work mathematically, they also set the context for purposeful computation.

Mental, written and calculator strategies can all be developed and practised through problems and investigations.

- Mental computation is the most important of the three in the primary school years, as it helps develop an understanding of computation processes and is fundamental to competent calculator use.
- Estimation is one of the keys to successful mental calculation. Quite often an estimate is all that is required and mental calculation should be the means of making an estimate. Exact answers should first be estimated, for the purpose of ensuring the final answer makes sense in terms of the problem, then calculated mentally or with calculators or computers.

- One important role of written computation is to assist mental computation. Sometimes it is useful to record subtotals on the way to finding an answer mentally. Standard written algorithms can be useful in helping to understand why certain answers and patterns occur. For example, if eleven multiplied by eleven is calculated in writing it is quite clear why 121 is the answer.
- Calculators are powerful tools for investigating important mathematical ideas, and for assisting students to learn mental and written computation concepts and strategies. For example, if children are learning to use a standard algorithm, they can use a calculator to check their answers. If their answers are correct, they can be confident they are using the algorithm correctly. If their answers are incorrect the calculator answer may help them see where they went wrong.

Principles of Mathematics Teaching

The following principles underpin the approach to teaching mathematics outlined in this book.

1 A mathematics program is concerned essentially with each student's progress as a mathematician.

Mathematicians are enthused and fascinated by problems, puzzles and patterns. They are competent problem-solvers and investigators who confidently use a variety of skills and draw on a wide range of knowledge.

2 Problems and investigations are central to a mathematics program.

As students pose and solve problems and investigate mathematical topics, they learn mathematics in a real context so that mathematics makes sense to them. Within the context of problems and investigations, students gain knowledge and skills and apply them purposefully.

3 Teachers use the problems posed by students as starting points for further work.

Students are very interested in the problems their friends pose and they like to challenge each other with increasingly complex problems. Each student can write problems that draw on their own background experiences and understandings, and this is one way a mathematics program is inclusive of all students.

4 There is no one way to solve a problem; students work out their own ways to find answers.

With the support of teachers who facilitate learning, rather than directly teaching techniques for working out answers, students can learn a wide range of problem-solving strategies through investigation and experimentation in cooperation with their friends.

5 Students build an understanding of mathematical ideas through many practical investigations and through talking with their teachers and friends about their observations and ideas.

In mathematics lessons students might be cutting and pasting, drawing and painting, building models, making patterns, playing games and explaining and discussing ideas. Teachers have a major role to play in helping students move from concrete experiences, which they discuss in their own language, to the symbolic expression of mathematical ideas.

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6 Teachers primarily use the language the students spontaneously use and gradually introduce them to more sophisticated terminology as they find a need for it.

By using the students' own words, we make a significant contribution to their growing understanding of mathematics and their progress along the path towards symbolic expression. Students should do a lot of talking and writing in mathematics lessons. They discuss their ideas, write problems and write reports on their investigations of mathematical topics. They will learn the standard terminology and symbols of mathematics so that they can communicate mathematical ideas effectively.

7 Teachers help students learn skills and acquire knowledge as outcomes of their work with problems and investigations.

Learning skills and acquiring knowledge should not be ends in themselves, their acquisition should be purposeful. Problem solving and investigation provide a purposeful context.

8 Teachers are particularly concerned that students develop the skills of visualisation, estimation and mental calculation.

These three skills areas, essential to effective problem-solving and investigation, are reciprocally linked with a knowledge of number facts: as students visualise, estimate and mentally calculate, they build a bank of number facts, and as they acquire number fact knowledge, they enhance their ability to work out answers and discover patterns through visualisation, estimation and mental calculation.

9 Teachers monitor each student's progress.

By compiling a profile of each student's growth as a mathematician, using anecdotal records, interview notes and work samples, teachers can monitor each student's progress. Students can be closely involved in monitoring their own progress.