Posing and Solving Problems

Organising Problem Posing

Throughout the primary school years, problems are central to a mathematics program, particularly problems that students themselves pose. The students' problems may be real or contrived, as long as they are posed in the spirit of mathematical exploration. Children like to challenge each other and test out their new understandings with their problems, and they are usually keen to solve the problems set by their peers. Having students pose problems is also a useful evaluation strategy as, through their problems, they reveal to teachers their mathematical understandings and misconceptions.

Teachers also have a role to play in posing problems, primarily: to model possibilities, ensure students meet a wide range of problem types and focus on understandings, knowledge and skills across a diversity of mathematical topics. There are several ways problem posing can be organised for middle and upper primary students. The following three ways have been successful.

Posing problems for middle and upper primary students

- A Personal Book for writing problems from which they read at a class or group problem sharing time.

- **Cards** which can be filed for use at an appropriate time; the problems can be sorted easily if necessary, as sometimes it is useful to extract a particular concept from a collection of problems.
- Hang Sheets of Paper on a Classroom Wall and ask students to record their problems in a felt-tip pen. The teacher can model writing problems on the paper and suggest to students that they can add problems to the collection.

The following problems appeared in a Year 5/6 class over a fortnight. They were written by the children on a long sheet of paper pinned to the classroom wall. Note the variety and the wealth of starting points for work. Note also the sources of the problems; some students have written problems similar to ones met in previous mathematics classes, some have presented real problems, some have posed problems as an outcome of investigative work and some have been inspired to offer original problems.

I bought a piece of pizza costing 25c for one eighth of the pizza. The whole pizza costs $1.50. Would it be cheaper to buy the whole pizza or one eighth at a time?

If Johnny had 6449 marbles and had to share them amongst 7 people, how many marbles would there be left over?

Sue went shopping and bought a pair of jeans that cost $40.95, 3 Mars Bars that cost 63c each and a present for her Mum that cost $15.62. How much change did she have if she took $65 with her?

If there were 1,080 children in a school, how many staff would the school need?

Look at this pattern:
98-89=9 87-78=9 76-67=9 65-56=9
54-45=9 43-34=9 32-23=9 21-12=9
10-01=9 How can this be?

Try and make more than 235 words on your calculator.

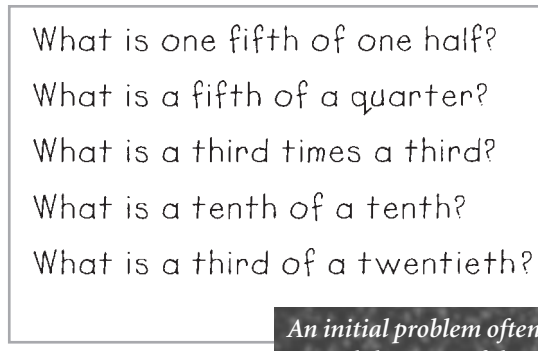
A Swiss skier died when he was practising speed skiing before his Olympic race. The slope was 70% gradient. How many degrees are in the angle at the bottom of the slope?

We are making models of Jupiter and Venus. Jupiter's diameter is 142,800 km and Venus' diameter is 12,100 km. We are using a scale of 1 cm: 2,000 km. How big will our models be? [Another child added the next question.] What about the sun?

1992 is the 16th Winter Olympics. They are on every four years so when were the first ones on? Listen to the TV and see if you are right.

A typical range of real and theoretical problems posed by Year 5/6 students.

Often, when a student poses a type of problem new to their classmates, others respond by posing similar problems or extending the scope of the original problem. Sometimes students engage in an exchange of problems exploring a concept, developing each other's ideas and setting challenges. The following sequence of problems is an example of this. One girl wrote the first problem and, as others found they enjoyed the challenge of working on it, they set the later problems.



An initial problem often prompts other students to extend the scope of the original investigation.

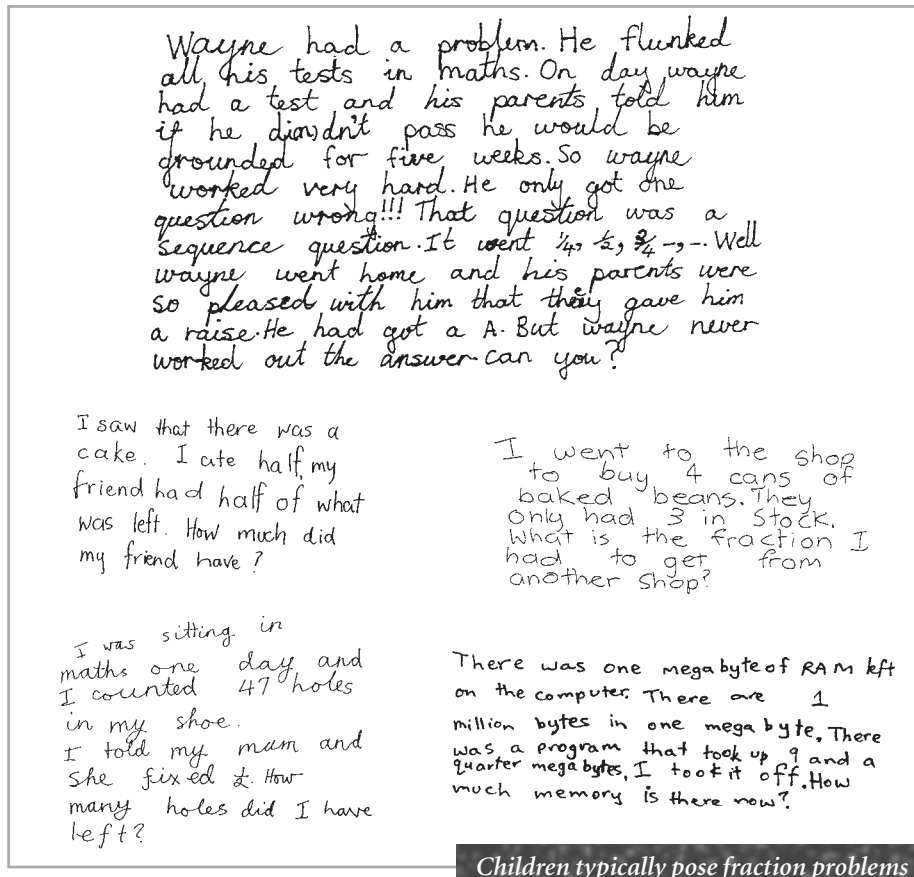
To add variety to the task of posing problems, a teacher can occasionally ask students to suggest a theme for their problems. Some examples teachers and children have suggested are: animal statistics, sports, Christmas, countries of the world, maps, food, temperatures, holidays and cars.

Another successful exercise is to take students into particular settings, such as the school playground, to search for possible problems. The outdoors has proved to be a rich source of ideas for problems related to school buildings, playground structures, car park statistics, shadow lengths, sports field measurements, playground usage statistics, distances, shape and measurement.

Posing Problems on Specific Mathematical Topics

Teachers can sometimes challenge students to pose problems on a particular mathematical topic. This can be an excellent way to introduce a specific topic of work, sometimes drawn from a set syllabus. For instance, students in one class were asked to write problems involving fractions. After the problems had been written, data was collected on the fractions they had used. Most had posed problems involving half, one-quarter or three-quarters and a few had used thirds. Only one child had used other types of fractions and this fact was the focus of discussion about the most commonly used fractions in real life, and where and when fractions are useful.

Children in a Year 4/5 class wrote the following fraction problems.



Children typically pose fraction problems involving halves and quarters.

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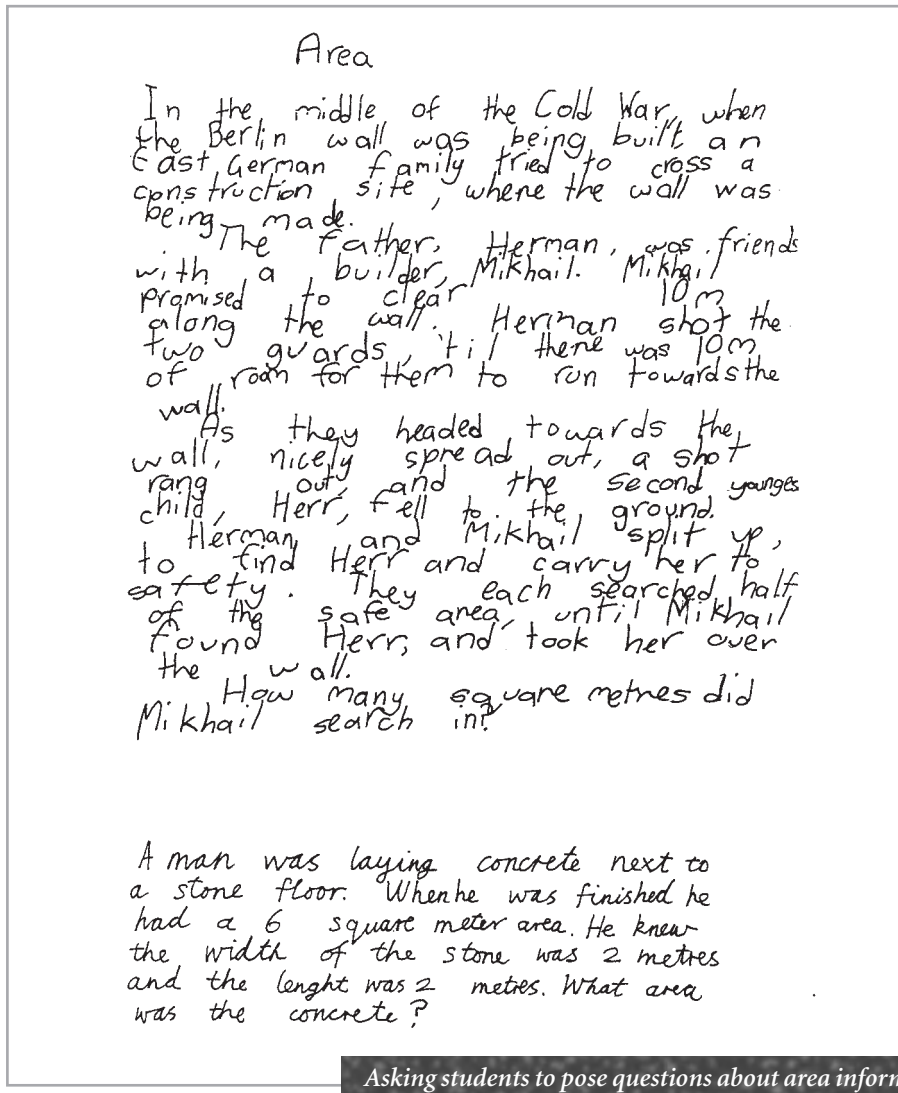
The problem about the holes in the shoe is an example of a 'trick' question, something that arises occasionally and delights the students.

When beginning a study of area, a class of Year 5/6 students was set the task of writing problems about area. This helped the teacher know something of the students' understandings and knowledge about area, and provided some starting points for work. Here are some of the problems that were produced.



Area

In the middle of the Cold War, when the Berlin wall was being built an East German family tried to cross a construction site, where the wall was being made.
The father, Herman, was friends with a builder, Mikhail. Mikhail promised to clear 10m along the wall. Herman shot the two guards, til there was 10m of room for them to run towards the wall.
As they headed towards the wall, nicely spread out, a shot rang out and the second youngest child, Herr, fell to the ground.
Herman and Mikhail split up, to find Herr and carry her to safety. They each searched half of the safe area, until Mikhail found Herr, and took her over the wall.
How many square metres did Mikhail search in?

A man was laying concrete next to a stone floor. When he was finished he had a 6 square meter area. He knew the width of the stone was 2 metres and the lenght was 2 metres. What area was the concrete?

Asking students to pose questions about area informs the teacher of the level of the students' understandings.

A flag that is 2m by 1·5m has an area of 3m².
If you doubled its length and width, what would the area be?

If you had a 36 cm^2 square and you coloured in a quarter of it how much area was left of the bit of square that wasn't coloured in?

My mum and dad were going to recarpet the living room floor. It was a rectangular room that was 3m by 5m and the carpet cost $18.50 per square metre. What is the area of the room and how much would it cost?

Some problems posed on the topic of area had a practical basis.

Scaffolding Problem Writing

The girl who wrote the coloured square problem was one of several who struggled with the wording of the problem. For students who have difficulty expressing their thoughts in writing, it is important that the teacher sits with them and asks them to work out their problems orally. This is also where the teacher needs to particularly focus on students from a non-English speaking background.

In the case of the area problems, the teacher asked the students having difficulties to draw their problem first, explain in steps what they had done in their drawings, and pose a question at the end. John produced the following problem as an outcome of this process.

I ruled up a 10cm X 10cm square. I coloured a fifth of it. How big is the area that is not coloured in?

John's written problem was based on his flow chart diagrams.

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Sharing Students' Problems

Students and teachers should share their problems regularly. They may be shared with the whole class or with a smaller group, or they may be made available for others to read at their leisure. Sharing their problems enhances students' communication skills as they strive to make their problems unambiguous. It also gives a purpose to the writing of problems.

The teacher's role in sharing times is to encourage and support students to use initiative in devising, explaining and refining their own problem solving strategies, including their own techniques for estimating, adding, multiplying, dividing and subtracting numbers. A regular time could be set aside each day for sharing problems.

In one Year 4 classroom, the students come into class after the recess break and immediately begin writing problems in personal problem books. As they finish, they join the teacher and their classmates on the floor to share some of the problems that have been written. Students form a circle and use MAB blocks as the basic material for supporting mental computation to work out the problems. One child has control of the MAB ones, another the tens, another the hundreds and another the thousands. Sometimes individuals use calculators or pencil and paper, and sometimes the teacher leads the children to use a large sheet of paper to do written calculations.

An alternative arrangement is for the teacher to suggest a time for discussion of particular problems so that students know they have to give them some consideration beforehand. With problems that have been written on sheets on the wall for all to see clearly, a proposed discussion time can be recorded against problems.

If, for example, 'Thursday, 10 a.m.' is written against a group of the children's problems, the children can spend time before then working alone or in a small group to solve the problems. When discussion time comes around the children can all gather together to share the work they have done.

I have used this method very successfully with upper primary classes whose daily timetable includes some 'personal study time' during which they can choose to work on problems.